Mathematics Department

Algebra 2

Developed By: Jessica Mabel, Leann Martin
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Scope and Sequence

Month	Algebra 2	Advanced Algebra 2	Accelerated Algebra 2
September	**Prerequisites** -Linear equations and inequalities -System of equations and inequalities - Order or Operations -Quadratic Equations -Functions **Prerequisites Test** **Quadratic Functions** - Graph quadratic functions. - Factor quadratic expressions and solve quadratic equations by factoring. - Find the zeros of a quadratic function by factoring. - Solve quadratic equations by taking the square root of both sides.	**Prerequisites** -Linear equations and inequalities -System of equations and inequalities - Order or Operations -Quadratic Equations -Functions **Prerequisites Test** **Quadratic Functions** - Graph quadratic functions. - Factor quadratic expressions and solve quadratic equations by factoring. - Find the zeros of a quadratic function by factoring. - Solve quadratic equations by taking the square root of both sides. - Perform operations on complex numbers. - Solve quadratic equations by completing the square. - Use completing the square to write quadratic functions in vertex form $y = a(x - h)^2 + k$. - Solve quadratic equations using the quadratic formula. - Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving. - Write and graph quadratic functions in standard form ($y = ax^2$ +	**Prerequisites** -Linear equations and inequalities -System of equations and inequalities - Order or Operations -Quadratic Equations -Functions **Prerequisites Test** **Quadratic Functions** - Graph quadratic functions. - Factor quadratic expressions and solve quadratic equations by factoring. - Find the zeros of a quadratic function by factoring. - Solve quadratic equations by taking the square root of both sides. - Perform operations on complex numbers. - Solve quadratic equations by completing the square. - Use completing the square to write quadratic functions in vertex form $y = a(x - h)^2 + k$. - Solve quadratic equations using the quadratic formula. - Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving. - Write and graph quadratic functions in standard form ($y = ax^2$ +

		bx + c), vertex form ($y = a(x - h)^2 + k$), and intercept form ($y = a(x - p)(x - q)$).	bx + c), vertex form ($y = a(x - h)^2 + k$), and intercept form ($y = a(x - p)(x - q)$). - Solve quadratic inequalities in one variable by graphing and algebraically. -Graph quadratic inequalities in two variables to show all solutions. -Use quadratic equations, inequalities, and functions to model and solved real-life problems.
October	**Quadratic Functions (Cont)** - Perform operations on complex numbers. - Solve quadratic equations by completing the square. - Use completing the square to write quadratic functions in vertex form $y = a(x - h)^2 + k$. - Solve quadratic equations using the quadratic formula. - Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving.	**Quadratic Functions (Cont)** - Solve quadratic inequalities in one variable by graphing and algebraically. -Graph quadratic inequalities in two variables to show all solutions. -Use quadratic equations, inequalities, and functions to model and solved real-life problems. **Polynomials** -Apply the properties of exponents to simplify and evaluate expressions involving powers. -Evaluate a polynomial function. -Graph a polynomial function. -Add, subtract, and multiply polynomials. -Factor polynomial expressions. -Use factoring methods to solve polynomial equations. -Divide polynomials using long and synthetic division. -Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.	**Polynomials** -Apply the properties of exponents to simplify and evaluate expressions involving powers. -Evaluate a polynomial function. -Graph a polynomial function. -Add, subtract, and multiply polynomials. -Factor polynomial expressions. -Use factoring methods to solve polynomial equations. -Divide polynomials using long and synthetic division. -Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems. -Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. -Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. -Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function. -Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph. -Use polynomial equations and functions to model and solve real-life problems.
November	**Quadratic Functions (Cont)** -Write and graph quadratic functions in standard form ($y = ax^2 + bx + c$), vertex form ($y = a(x - h)^2 + k$), and intercept form ($y = a(x - p)(x - q)$). - Solve quadratic inequalities in one variable by graphing and algebraically. -Graph quadratic inequalities in two variables to show all solutions. -Use quadratic equations, inequalities, and functions to model and solved real-life problems. **Polynomials**	**Polynomials (Cont)** -Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. -Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. -Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function. -Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph. -Use polynomial equations and functions to model and solve real-life problems.	**Powers, Roots, and Exponents** -Evaluate *n*th roots of real numbers using both radical and rational exponent notation. -Apply the properties of exponents to simplifying and evaluating expressions containing rational exponents. -Perform operations with functions, including those that contain powers. -Find the inverse of both linear and non-linear functions. -Graph square root and cube root functions. -Solve equations that contain radicals and rational exponents. -Use radical and *n*th root equations

	-Apply the properties of exponents to simplify and evaluate expressions involving powers.	**Exponentials and Logs** -Graph exponential functions, including those that model growth and decay.	and functions to model and solve real-life problems.
December	**Polynomials (Cont)** -Evaluate a polynomial function. -Graph a polynomial function. -Add, subtract, and multiply polynomials. -Factor polynomial expressions. -Use factoring methods to solve polynomial equations. -Divide polynomials using long and synthetic division. -Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.	**Exponentials and Logs (Cont)** -Graph exponential functions that contain the natural base *e*. -Evaluate and simplify exponential expressions with base *e.* -Evaluate and simplify logarithmic expressions. -Graph logarithmic functions. -Use the properties of logarithms. -Solve exponential and logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems.	**Exponentials and Logs (Cont)** -Graph exponential functions that contain the natural base *e*. -Evaluate and simplify exponential expressions with base *e.* -Evaluate and simplify logarithmic expressions. -Graph logarithmic functions. -Use the properties of logarithms. -Solve exponential and logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems.
January	**Polynomials (Cont)** -Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. -Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. -Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function. -Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph. -Use polynomial equations and functions to model and solve real-life problems.	**Rational Functions** -Graph rational functions. -Write and use equations for inverse and joint variations. -Multiply and divide rational expressions. -Add and subtract rational expressions. -Simplify complex fractions. -Solve rational equations. Use rational equations and functions to model and solve real-life problems. **Powers, Roots, and Exponents** -Evaluate *n*th roots of real numbers using both radical and rational exponent notation. -Apply the properties of exponents to simplifying and evaluating expressions containing rational exponents	**Rational Functions** -Graph rational functions. -Write and use equations for inverse and joint variations. -Multiply and divide rational expressions. -Add and subtract rational expressions. -Simplify complex fractions. -Solve rational equations. Use rational equations and functions to model and solve real-life problems. **Data Analysis and Statistics** -Use counting principles and permutations
February	**Powers, Roots, and Exponents** -Evaluate *n*th roots of real numbers using both radical and rational exponent notation. -Apply the properties of exponents to simplifying and evaluating expressions containing rational exponents. -Perform operations with functions, including those that contain powers.	**Powers, Roots, and Exponents (Cont)** -Perform operations with functions, including those that contain powers. -Find the inverse of both linear and non-linear functions. -Graph square root and cube root functions. -Solve equations that contain radicals and rational exponents. -Use radical and *n*th root equations and functions to model and solve real-life problems.	**Data Analysis and Statistics (Cont)** -Use combinations and binomial theorem -Find probabilities of events -Study and interpret probability distributions -Study normal distribution - Select and draw conclusions from samples -Determine the margin of error for a survey -Compare surveys, experiments, and observational studies.

March	**Powers, Roots, and Exponents (Cont)** -Find the inverse of both linear and non-linear functions. -Graph square root and cube root functions. -Solve equations that contain radicals and rational exponents. -Use radical and *n*th root equations and functions to model and solve real-life problems.	**Sequences and Series** -Write rules for arithmetic sequences and find sums of arithmetic series. -Write rules for geometric sequences and find sums of geometric series. -Find sums of infinite geometric series. -Use summation notation to write a series. -Evaluate and write recursive rules for sequences. -Use sequences and series to solve real-life problems.	**Sequences and Series** -Write rules for arithmetic sequences and find sums of arithmetic series. -Write rules for geometric sequences and find sums of geometric series. -Find sums of infinite geometric series. -Use summation notation to write a series. -Evaluate and write recursive rules for sequences. -Use sequences and series to solve real-life problems.
April	**Exponentials** -Graph exponential functions, including those that model growth and decay. -Graph exponential functions that contain the natural base *e*. -Evaluate and simplify exponential expressions with base *e.*	**Data Analysis and Statistics (Cont)** -Use combinations and binomial theorem -Find probabilities of events -Study and interpret probability distributions -Study normal distribution - Select and draw conclusions from samples -Determine the margin of error for a survey -Compare surveys, experiments, and observational studies.	**Conic Sections** -Determine the distance between two points. -Determine the midpoint of the line segment joining two points. -Graph and write equations of parabolas. -Graph and write equations of circles. -Graph and write equations of ellipses. -Graph and write equations of hyperbolas. -Classify a conic section using its equation. -Solve systems of quadratic systems algebraically and sketch the graph. -Use conics to model and solve real-life problems.
May	**Logs** -Evaluate and simplify logarithmic expressions. -Graph logarithmic functions. -Use the properties of logarithms. -Solve exponential and logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems**.**	**Trigonometric Ratios and Functions** -Describe and measure angles using radian and degree measure. -Convert degree measure to radian measure and vice versa. -Evaluate trigonometric functions of any angle, given a point on its terminal side. -Evaluate trigonometric functions of any angle, with or without using a reference angle. -Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) -Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. -Identify trigonometric functions of special angles with or without the use of a unit circle. -Identify and construct a unit circle and recognize its relationship to real numbers. -Use a calculator to approximately evaluate a trigonometric function of any angle. -Use the right triangle trigonometric	**Trigonometric Ratios and Functions** -Describe and measure angles using radian and degree measure. -Convert degree measure to radian measure and vice versa. -Evaluate trigonometric functions of any angle, given a point on its terminal side. -Evaluate trigonometric functions of any angle, with or without using a reference angle. -Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) -Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. -Identify trigonometric functions of special angles with or without the use of a unit circle. -Identify and construct a unit circle and recognize its relationship to real numbers. -Use a calculator to approximately evaluate a trigonometric function of any angle. -Use the right triangle trigonometric

		ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles. -Use the Law of Sines to solve oblique triangles. -Use the Law of Cosines to solve oblique triangles. -Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.	ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles. -Use the Law of Sines to solve oblique triangles. -Use the Law of Cosines to solve oblique triangles. -Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.
June	**Rational Functions** -Graph rational functions. -Write and use equations for inverse and joint variations. -Multiply and divide rational expressions. -Add and subtract rational expressions. -Simplify complex fractions. -Solve rational equations. Use rational equations and functions to model and solve real-life problems.	**Conic Sections** -Determine the distance between two points. -Determine the midpoint of the line segment joining two points. -Graph and write equations of parabolas. -Graph and write equations of circles. -Graph and write equations of ellipses. -Graph and write equations of hyperbolas. -Classify a conic section using its equation. -Solve systems of quadratic systems algebraically and sketch the graph. -Use conics to model and solve real-life problems.	**Trigonometric Graphs, Identities, and Equations** -Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. -Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph. -Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts). -Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions. -Use the Fundamental Trigonometric Identities to verify other trigonometric identities. -Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions. -Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions. -Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.

Unit 1

Quadratics

Summary and Rationale

Unit 1 extends the study of functions to quadratic functions. Students will thoroughly analyze the characteristics and properties of quadratic functions, their equations and graphs. Students will learn different solving and graphing methods, as well as how to write and obtain important information from quadratic equations in different forms. During this unit, it is crucial for students to see the relationship between the *solutions* of a quadratic equation, the *zeros* of a quadratic function, and the *x-intercept's* of this function's graph. This connection will follow through with other types of functions that will be studied in future units. The graph of a quadratic function is called a parabola, which is one of the four conic sections that will be studies in Unit 10. Students will also solve and graph one and two variable quadratic inequalities

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.CN.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a +bi with a and b real.
N.CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N.CN.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
N.CN.7	Solve quadratic equations with real coefficients that have complex solutions
N.CN.8	Extend polynomial identities to the complex numbers
N.CN.9	Know the fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Algebra	

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. k. Interpret parts of an expression, such as terms, factors, and coefficients. l. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. g. Factor a quadratic expression to reveal the zeros of the function it defines. h. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. i. Use the properties of exponents to transform expression for exponential functions.
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
A.REI.4	Solve quadratic equations in one variable. c. Use the method of completing the square to transform any quadratic equations in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. d. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equations. Recognize when the quadratic formula gives complex solutions and write them as a + bi and a – bi for real numbers a and b.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena. Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.	What does the equation and graph of a quadratic function look like? What is the reason for having the different methods for solving quadratics and when can using one method be necessary or more appropriate than another?

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

Quadratic equations are of degree two, therefore they have two solutions that can be determined from a graph (The solutions are the x-intercepts.), or by using different algebraic methods. Four algebraic methods are factoring, taking the square root of both sides, completing the square, or using the Quadratic Formula. The best method to use depends on the forms and characteristics of the given equations, the nature of the solutions, and how precise the solutions should be.

Quadratics can have real number solutions, but can also have solutions in a larger system, called the complex numbers. There are differences between the results of operations on complex numbers from those obtained within the real number system.

The graph of a quadratic function in a coordinate plane is a u-shaped graph, called a parabola. Using algebraic methods to manipulate and/or solve the equation of a quadratic function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

The *solutions* of a quadratic equation $ax^2 + bx + c = 0$ are equal to the *real zeros* of the related quadratic function $y = ax^2 + bx + c$, and the *x-intercepts* of this function's graph. Zeros that are not real are not visible on the graph of a function in a coordinate plane of real numbers.

Quadratic inequalities can be one variable or two, and both types can be solved and represented algebraically and graphically.

How does using the different forms of a quadratic function help us graph?

What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs?

$$y = a(x - h) + k$$
$$y = a|x - h| + k$$
$$y = a(x - h)^2 + k$$

How do the results of operations on real numbers differ when applied to the complex numbers?

What are the connections between the algebraic and graphical representations of both one-variable and two variable quadratic inequalities?

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Quadratics · Quadratic equations · Inequalities · Graphing	· Graph quadratic functions. · Factor quadratic expressions and solve quadratic equations by factoring. · Find the zeros of a quadratic function by factoring. · Solve quadratic equations by taking the square root of both sides. · Perform operations on complex numbers. ·Solve quadratic equations by completing the square. · Use completing the square to write quadratic functions in vertex form $y = a(x - h)^2 + k$. ·Solve quadratic equations using the quadratic formula. ·Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving. · Write and graph quadratic functions in standard form ($y = ax^2 + bx + c$), vertex form ($y = a(x - h)^2 + k$), and intercept form ($y = a(x - p)(x - q)$). ·Solve quadratic inequalities in one variable by graphing and algebraically. ·Graph quadratic inequalities in two variables to show all solutions. ·Use quadratic equations, inequalities, and functions to model and solved real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract

-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

LINKS TO CAREERS: https://www.math.uci.edu/math-majors/math-career-resources

Unit 2

Polynomials

Summary and Rationale

Unit 2 extends the study of functions to polynomial functions. Students will first use the Properties of Exponents to simplify and perform operations involving powers, monomials and polynomials. Students will then thoroughly analyze the characteristics and properties of polynomial functions, their equations and graphs. The quadratic functions studied in Unit 5 are polynomial functions of degree two. Students will learn different solving and graphing methods, as well as how to write and obtain important information from polynomial equations of degree greater than two in different forms. During this unit, it is crucial for students to revisit the relationship between the *solutions* of a polynomial equation, the *zeros* of a polynomial function, and the *x-intercept's* of this function's graph.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.CN.8	Extend polynomial identities to the complex numbers.
N.CN.9	Know the fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the

	relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.1	Write a functions that describes a relationship between two quantities. d. Determine an explicit expression, a recursive process, or steps for calculation from a context. e. Combine standard functions types using arithmetic operations. d. Compose functions
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. m. Interpret parts of an expression, such as terms, factors, and coefficients. n. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.2	Know and apply the remainder Theorem: For a polynomial p(x) and a number a, the remainder of division by x – a is p(a) = 0 if and only if (x – a) is a factor of p (x)
A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomials.
A.APR.4	Prove polynomial identities and use them to describe numerical relationships.
A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Performing operations on polynomial expressions requires the accurate application of the properties of exponents and the order of operations, as well as the ability to distinguish between like and unlike terms. The requirements for and the results of addition and subtraction of polynomials differ greatly from those involving multiplication, division, and powers. A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.	What are the requirements for the addition and subtraction of monomials and polynomials? Are these the same for multiplication and division? How do the results of the operations differ? What steps should be taken when factoring a polynomial expression or equation? What specifics do you look for and how do you know when it is completely factored? What characteristics, other than its solutions, of a polynomial equation can be used to describe the behavior and sketch the graph of its related function?

Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

All equations of degree one or higher are defined as polynomial equations. The Fundamental Theorem of Algebra states that the number of solutions to a one-variable polynomial equation is equal to the degree of the polynomial. These solutions can be determined through graphing (The solutions are the x-intercepts.) and using one more previously learned and new algebraic methods and theorems. The best method(s) to use depends on the forms and characteristics of the given equations, the nature of the solutions, and how precise the solutions should be.

Polynomials can have real number solutions, but can also have solutions in a larger system, called the complex numbers. There are differences between the results of operations on complex numbers from those obtained within the real number system.

The graphs of polynomial functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a polynomial function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

The *solutions* of a polynomial equation are equal to the *real zeros* of the related polynomial function and the *x-intercepts* of this function's graph. Zeros that are not real are not visible on the graph of a function in a coordinate plane of real numbers.

What theorems and methods are used to identify the total number of roots and determine the real zeros of a polynomial function?

Which types of roots are seen on the graph of a polynomial function? Which are not visible? What happens in the graph if there is a double root?

What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs?

- $y = a\,(x - h) + k$
- $y = a\,|\,x - h\,| + k$
- $y = a\,(x - h)^2 + k$
- $y = a\,(x - h)^n + k$

Evidence of Learning (Assessments)

Tests
Quizzes
Homework

Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Polynomials · Fundamental Theorem of Algebra · Rational Zero Theorem · Graphing	· Apply the properties of exponents to simplify and evaluate expressions involving powers. · Evaluate a polynomial function. · Graph a polynomial function. · Add, subtract, and multiply polynomials. · Factor polynomial expressions. · Use factoring methods to solve polynomial equations. · Divide polynomials using long and synthetic division. · Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems. · Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. · Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. · Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function. · Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph. · Use polynomial equations and functions to model and solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling

-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

LINKS TO CAREERS: https://www.math.uci.edu/math-majors/math-career-resources

Unit 3

Powers, Roots, and Radicals

Summary and Rationale

Unit 3 connects and extends the familiar ideas of squares and square roots from Algebra 1 to other exponents and roots, including those that are *not* whole numbers in expressions and equations. The unit also extends the study of functions to radical and root functions.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.

	b. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a) Factor a quadratic expression to reveal the zeros of the function it defines. b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c) Use the properties of exponents to transform expression for exponential functions.
A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Functions	
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.4	Find inverse functions. a) Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b) Verify by composition that one function is the inverse of another. c) Read values of an inverse function from a graph or table, given that function has an inverse. d) Produce an invertible function from a non-invertible function by restricting the domain.
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Radical expressions can be rewritten using rational exponents and vice versa. The Properties of Exponents apply to all rational exponents.	How are the graphs of a function and its inverse related? What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs? • $y = a\,(x - h) + k$

A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.

Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

The graphs of radical functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a radical function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

Functions can be added, subtracted, multiplied, or divided to form a new function. They can also be combined or repeated to form a new function, called a composite function. The new functions created may have different domains and ranges than their parts.

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line $y = x$.

The inverse of a power function of n^{th} degree is an n^{th} root function. The n^{th} degree functions that are classified as even functions require domain restrictions.

The use of inverse operations and the properties of equality can be applied to the solving of radical equations and equations with rational exponents. However, certain procedures may lead to invalid solutions called extraneous solutions (or roots).

- $y = a \mid x - h \mid + k$
- $y = a\,(x - h)^2 + k$
- $y = a\,(x - h)^n + k$
- $y = \sqrt[n]{(x - h)} + k$

When and how do you check for extraneous solutions when solving equations containing radicals and rational exponents?

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Powers · Roots · Radicals · Exponents	· Evaluate *n*th roots of real numbers using both radical and rational exponent notation. · Apply the properties of exponents to simplifying and evaluating expressions containing rational exponents. · Perform operations with functions, including those that contain powers. · Find the inverse of both linear and nonlinear functions. · Graph square root and cube root functions. · Solve equations that contain radicals and rational exponents. · Use radical and *n*th root equations and functions to model and solve real-life problems

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

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Unit 4

Exponents and Logarithms

Summary and Rationale

Unit 4 extends the study of functions to exponential and logarithmic functions, which are two important families of functions because they model many real-life situations. Students will analyze the characteristics and properties these functions, their equations and graphs. Students will explore graphs that contain asymptotes as they draw and recognize patterns within the behavior of exponential and logarithmic functions. Previously learned Properties of Exponents and new Properties of Logarithms will be used to simplify expressions and solve equations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. c. Interpret parts of an expression, such as terms, factors, and coefficients. d. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. d) Factor a quadratic expression to reveal the zeros of the function it defines. e) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. f) Use the properties of exponents to transform expression for exponential functions
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a) Graph linear and quadratic functions and show intercepts, maxima, and minima. b) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e) Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b) Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.5	Understand that inverse relationships between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a) Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit

	interval relative to another

Interdisciplinary Connections
Standard x.x

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena. Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range. A relationship between two variables or two sets of data is an exponential function if the two variables increase (grow) or decrease (decay) by the same *percent* over equal periods of time. The inverse of an exponential function of a logarithmic function of the same base. Natural exponential and logarithmic functions are of the natural base, *e*. Exponential functions have restrictions on the range, therefore logarithmic functions contain domain restrictions. The graphs of these functions contain asymptotes, which are arbitrary lines that a graph approaches as you move away from the origin. Every function has an inverse, however the inverse is not always another function. There is an algebraic	What values of b does $y = b^x$ represent exponential growth? Decay? How do you determine the asymptotes of an exponential or logarithmic graph? What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs? ● $y = a\,(x - h) + k$ ● $y = a\,\lvert x - h \rvert + k$ ● $y = a\,(x - h)^2 + k$ ● $y = a\,(x - h)^n + k$ ● $y = \sqrt[n]{(x - h)} + k$ ● $y = ab^{(x-h)} + k$ ● $y = \log_b (x - h) + k$ How can you use inverse functions to graph logarithmic functions? How is solving a logarithmic equation similar to solving an exponential equation? How is it different? Why do logarithmic equations sometimes contain extraneous solutions?

procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line y = x. The graphs of exponential and logarithmic functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a exponential or logarithmic function can throw light on the function's properties such as its zeros and asymptotes, and can also help visualize the behavior of its graph. These strategies can result in the use of more effective and efficient graphing methods. The use of inverse operations with the properties of equality, exponents, and logarithms can be applied to the solving of exponential and logarithmic equations. However, since the domains of logarithmic functions contain restrictions, obtaining extraneous solutions is a possibility.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know: · Exponents · Logarithms · Graphing	Students will be able to: · Graph exponential functions, including those that model growth and decay. · Graph exponential functions that contain the natural base *e*. · Evaluate and simplify exponential expressions with base *e.* · Evaluate and simplify logarithmic expressions. · Graph logarithmic functions. · Use the properties of logarithms. · Solve exponential and logarithmic equations. · Use exponential and logarithmic functions to model and solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

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Unit 5

Rational Equations and Functions

Summary and Rationale

Unit 5 extends the study of functions to rational functions, which is the ratio of two polynomial functions. Students will analyze the characteristics and properties of these functions, their equations and graphs. Students will explore functions that contain domain and range restrictions as they draw graphs that contain asymptotes and other forms of discontinuity. Familiar concepts involving simplifying and performing operations on rational numbers will be extended to operations of rational expressions. Strategies for solving equations will be extended and applied to solving rational equations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. e. Interpret parts of an expression, such as terms, factors, and coefficients. f. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.

A.APR.6	Rewrite simple rational expressions in different forms; write a(x) / b(x) in the form q(x) + r(x) / b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.7	Understand that rational expressions form a system of analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. f) Graph linear and quadratic functions and show intercepts, maxima, and minima. g) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. h) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. i) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. j) Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude

F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. c) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. d) Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena. Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range. A rational function is the ratio of two polynomial functions. These functions contain restrictions on their domains and/or ranges. Therefore, their graphs contain asymptotes, holes, and/or discontinuity. The graphs of rational functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a rational function can throw light on the function's properties such as its zeroes, asymptotes, domain,	How can you tell whether a set of data pairs (x, y) shows inverse variation? How do you determine the vertical and horizontal asymptotes for the graph of a rational function? What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs? • $y = a\,(x - h) + k$ • $y = a\,\lvert x - h \rvert + k$ • $y = a\,(x - h)^2 + k$ • $y = a\,(x - h)^n + k$ • $y = \sqrt[n]{(x - h)} + k$ • $y = ab^{(x-h)} + k$ • $y = log_b\,(x - h) + k$ • $y = \frac{a}{(x-h)} + k$ How is adding (or subtracting, multiplying, dividing simplifying) rational expressions similar to adding (or subtracting, multiplying, dividing, simplifying) numerical fractions?

range, and discontinuity, which all can also help visualize the behavior of its graph. These strategies can result in the use of more effective and efficient graphing methods.

The previously learned procedures for multiplying, dividing, simplifying, adding, and subtracting numerical fractions are extended to performing the same operations on algebraic rational expressions.

The use of inverse operations, the properties of equalities, and cross multiplication can be applied to the solving of rational equations. Any solutions obtained that are not within the domain of the related function are extraneous.

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Rational equations · Functions · Inverse variations	· Graph rational functions. · Write and use equations for inverse and joint variations. · Multiply and divide rational expressions. · Add and subtract rational expressions. · Simplify complex fractions. · Solve rational equations. · Use rational equations and functions to model and solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

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Unit 6

Conic Sections

Summary and Rationale

Unit 6 involves the study of the four conic sections, parabolas, circles, ellipses, and hyperbolas. Conic sections are an important part of the study of algebra and geometry because they have many different real-life applications.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. a) Interpret parts of an expression, such as terms, factors, and coefficients. b) Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. g) Factor a quadratic expression to reveal the zeros of the function it defines. h) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. i) Use the properties of exponents to transform expression for exponential functions
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Geometry	
G.GPE.1	Derive the equations of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and the radius of a circle given by an example.
G.GPE.2	Derive the equation of a parabola given a focus and directrix.
G.GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of the distances from the foci is constant.
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
The Distance Formula is an application of the previously learned Pythagorean Theorem. A conic section is a graph of an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ These relations are studied in algebra because they have various real-life applications. The only conic sections that are functions are parabolas that open upward or downward, previously learned as quadratic functions and hyperbolas that are written in the form of a rational function. Using algebra to manipulate the equation of a conic section, particularly the method of "completing the square" can be used to determine the parts and properties of its graph, and can result in the use of more effective and efficient graphing methods. Two quadratic equations form a quadratic system. The solutions of this type of system are represented by the	Given the equation of a conic section in the following form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ how do you classify its graph? How do you rewrite the equation so that you can obtain the information needed to draw its graph? Which conic sections are functions? Which are relations, but not functions? How many solutions are possible for a system containing a conic section and a line? For a system of two conic sections? What would each possible solution case look like graphically? What algebraic methods can be used to solve a system of equations that contains one or more quadratic relation?

set of ordered pairs that satisfy both equations in the system. The solutions of these systems can be determined and represented both algebraically and graphically, using the same algebraic methods previously used to solve linear systems. However, solving quadratic systems requires methods learned for solving quadratic equations, and may yield more solutions.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Conic sections · Graphing·	· Determine the distance between two points. · Determine the midpoint of the line segment joining two points. · Graph and write equations of parabolas. · Graph and write equations of circles. · Graph and write equations of ellipses. · Graph and write equations of hyperbolas. · Classify a conic section using its equation. · Solve systems of quadratic systems algebraically and sketch the graph. · Use conics to model and solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

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21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
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- ❑ Use technology to enhance productivity.

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Unit 7

Sequences and Series

Summary and Rationale

Students have studied number patterns (or sequences) since elementary school. Unit 7 will connect this familiar exploration of number patterns to algebra as students write and use rules for sequences and series. An arithmetic sequence has a common difference, so it is similar to the linear functions. A geometric sequence has a common ratio, so it is similar to the exponential functions.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Algebra	
A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
Functions	
F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between two forms.
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Algebra can be used to write and use rules for number patterns that exist in sequences and series. A sequence is a list of terms that demonstrate a number pattern, while a series is an expression formed by adding the terms of the sequence. Both can be either finite or infinite. An arithmetic sequence has a common difference, so it is similar to a linear function. A geometric sequence has a common ratio, so it is similar to an exponential function. A recursive sequence is one that uses one or more previous term(s) to obtain the next term.	What is the difference between a sequence and a series? What makes a sequence or series arithmetic? geometric? What information do you need to find a term of an arithmetic (or geometric) series? How do you know if a sum of an infinite geometric series exists? And if it does, what information is needed to determine the sum? How can summation (or sigma) notation be used to represent a series?

Evidence of Learning (Assessments)
Tests Quizzes Homework Class participation

Objectives (SLO)	
Students will know: · Sequences · Series	Students will be able to: · Write rules for arithmetic sequences and find sums of arithmetic series. · Write rules for geometric sequences and find sums of geometric series. · Find sums of infinite geometric series. · Use summation notation to write a series. · Evaluate and write recursive rules for sequences. · Use sequences and series to solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
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Unit 8

Data Analysis and Statistics

Summary and Rationale

In unit 6 students will use previous skills of calculating probability and measures of central tendency, interpreting and displaying a data set, and performing simulations to predict outcomes. They will also learn new ways to collecting data in appropriate ways, finding binomial probabilities, and analyzing data to include expected value and variance. These skills will help students run experiments or conduct surveys, calculate and report appropriate measures when analyzing data, and to for a solid foundation for students in advanced statistics.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Interdisciplinary Connections	
Standard x.x	

Instructional Focus	
Enduring Understandings:	**Essential Questions:**

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
. .	.

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Unit 9

Trigonometric Functions of Angles

Summary and Rationale

Unit 9 continues the study of trigonometry, with a specific focus on the study of angles and triangles. Trigonometry is closely tied to both algebra and geometry. Students began their study of trigonometry in their geometry courses with right triangles. This unit will provide opportunities for students to further explore the more complex and comprehensive ideas behind the use of trigonometry in triangle measurement. Students will find trigonometric functions of angles, solve triangles, and use trigonometry to calculate the area of triangles.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Functions	
F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4, and p/6 and use the unit circle to express the values of sine, cosine, and tangent for p - x, p + x and 2p - x in terms of their values for x, where x is any real number.
I.TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Geometry	
G.SRT.7	Explain and use the relationship between the side and cosine complementary angles
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.9	Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10	Prove the Laws of Sines and Cosines and use them to solve problems
G.SRT.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Trigonometric functions of acute angles are defined by ratios of side lengths of right triangles. These ratios are: sine, cosine, tangent, cosecant, secant, and cotangent.	How do you convert between degrees and radians? How can you use the trigonometric ratios to find the measurements of sides or angles of a right triangle?

The angles 30°, 45°, and 60° occur frequently in trigonometry. These are the angles of the two special right triangles previously learned in geometry (30°-60°-90° and 45°-45°-90°) The values of the six trigonometric functions of these angles do not need to be memorized since they can quickly be generated by drawing the two special right triangles and labeling the sides of each. Trigonometric functions and their inverses can be used to find a missing side length or angle measure in a right triangle. Finding all missing lengths and angle measures is called solving a right triangle. An angle is formed by two rays that have a common endpoint, called the vertex. You can generate an angle by fixing one ray, called the initial side, and rotating the other ray, called the terminal side, about the vertex. In a coordinate plane, an angle whose vertex is at the origin and whose initial side is the positive x-axis on in standard position. The phrase "the terminal side of θ lies in quadrant…" is simply saying that θ lies in that quadrant. Two angles in standard position are co-terminal if their terminal sides coincide. An angle co-terminal with a given angle can be found by adding or subtracting multiples of 360°. Angles can be measured in degrees and in radians. The measure of an angle is determined by the amount and direction of a rotation from the initial side to the terminal side. The angles measure is positive is the rotation is counterclockwise and negative if the rotation is clockwise. The terminal side of an angle can make more than one complete rotation. To define a radian, consider a circle with radius r centered at the origin. One radian is the measure of an angle in standard position whose terminal side intercepts the arc of length r. Since the circumference of a circle is equal to $2\pi r$, there are 2π radians in a circle. Therefore 2π radians is equivalent to 360°. (π radians = 180°, and so on…) The length of an arc and the area of a sector of a circle can be calculated using the central angle of the circle in degrees (previously learned), and in radians. Reference angles are used to evaluate trigonometric functions of *any* angle.	How can you use The Law of Sines and The Law of Cosines to solve any triangle?

The Law of Sines and The Law of Cosines can be used to solve *any* triangle. (Determine all missing side lengths of angle measurements.) The use of one law rather than the other depends on the known information about he triangle. Some problems may show cases in which a triangle cannot be made under the given conditions (No solution). Other cases yield one or two possible triangle solutions. Trigonometric ratios can also be used to calculate the area of *any* triangle.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Trigonometric functions · Law of Sines · Law of Cosines · Heron's Area Formula	· Describe and measure angles using radian and degree measure. · Convert degree measure to radian measure and vice versa. · Evaluate trigonometric functions of any angle, given a point on its terminal side. · Evaluate trigonometric functions of any angle, with or without using a reference angle. · Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) · Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. · Identify trigonometric functions of special angles with or without the use of a unit circle. · Identify and construct a unit circle and recognize its relationship to real numbers. · Use a calculator to approximately evaluate a trigonometric function of any angle. · Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles.

	· Use the Law of Sines to solve oblique triangles. · Use the Law of Cosines to solve oblique triangles. · Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.

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Unit 10

Trigonometric Functions of Real Numbers

Summary and Rationale

The focus of Unit 10 is the study of trigonometric functions of real numbers, used to model periodic behavior. Students will have opportunities to see more connections between trigonometry and algebra as they graph trigonometric functions and their inverses in a coordinate plane. Just as they have done with other types of functions in previous units, students will analyze the characteristics of their equations and graphs. Students will also use advanced algebra skills to evaluate trigonometric functions, verify trigonometric identities, and solve trigonometric equations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity

N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Algebra	
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Functions	
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F.TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.8	Prove the Pythagorean identity $\sin^2(q) + \cos^2(q) = 1$ and use it to find sin(q), cos(q), or tan(q) given sin(q), cos(q), or tan(q) and the quadrant of the angle.
F.TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	

G.SRT.7	Explain and use the relationship between the side and cosine complementary angles.
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Interdisciplinary Connections	
Standard x.x	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena. Periodic functions have graphs that have repeating patterns that continue indefinitely. The shortest repeating portion is called a cycle. The horizontal length of each cycle is called the period. Trigonometric functions are periodic functions. Determining an output value of a trigonometric function, given an input value, requires evaluating the trigonometric expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing	Why are trigonometric functions called periodic functions? What is a trigonometric identity? How can trigonometric identities be used to determine exact trigonometric values of uncommon angles? How can you solve trigonometric equations?

the relationship of the function, as well as its complete domain and range.

The graphs of the sine and cosine are periodic functions that both have the following characteristics: The domain of each function is all real numbers and the range of each function is $-1 \leq y \leq 1$. Each graph has a period of 2π (or 360°.)

The graph of tangent is a periodic function with a domain of all real numbers except odd multiples of $\pi/2$. At odd multiples of $\pi/2$, the graph has vertical asymptotes. The range is all real numbers. The graph has a period of π (or 180°.)

Families of functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of *a, h,* and *k* similarly affect their graphs:

§ $y = a(x - h) + k$
§ $y = a|x - h| + k$
§ $y = a(x - h)^2 + k$
§ $y = a(x - h)^n + k$
§ $y = \frac{a}{(x - h)} + k$
§ $y = ab^{(x-h)} + k$
§ $y = \log_b(x - h) + k$
§ $y = a \sin b(x - h) + k$
§ $y = a \cos b(x - h) + k$
§ $y = a \tan b(x - h) + k$

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line y = x.

Trigonometric and inverse trigonometric functions and their graphs can be used to model and solve real-world problems, particularly those that involve periodic behavior.

Trigonometric identities are equations that are true for all values of x in their domain. Examples of Fundamental Trigonometric Identities are: Reciprocal Identities, Tangent and Cotangent Identities, Pythagorean Identities, Co-function Identities, and Negative Angle Identities. These trigonometric identities can be used to evaluate trigonometric functions, simplify trigonometric expressions, and verify other identities.

A verification of a trigonometric identity is a chain of equivalent expressions showing that one side of the identity is equal to the other side. When verifying an identity a common and useful strategy is to begin with the expression from one side and manipulate it algebraically until it is identical to the other side.

Properties of real numbers and equality, along with the use of inverse operations and trigonometric identities, can transform a trigonometric equation into one or a series of equivalent simpler equations. These properties can be used repeatedly to isolate the variable. Algebraic techniques such as factoring or The Quadratic Formula may also be necessary during the process of finding solutions to trigonometric equations in quadratic form. Extraneous solutions can result if an equation is being solved within a restricted domain.

Sum, Difference, Double, and Half-Angle Formulas make it possible to determine exact trigonometric values for less common angles using the more common angles such as 30°, 45°, 60°, etc.

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Trigonometric functions of real numbers · Formulas · Graphing	· Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. · Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph. · Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts). · Evaluate, graph and identify the domains and ranges of inverse trigonometric functions. · Write equations for inverse trigonometric functions. · Evaluate compositions of trigonometric functions. · Use trigonometric functions and their inverses to model and solve real-life problems. · Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions. · Use the Fundamental Trigonometric Identities to verify other trigonometric identities. · Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions. · Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions. Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.

Suggested Resources/Technology Tools

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