Mathematics Department

Pre-Calculus

Developed By: Jessica Mabel, Leann Martin, Mark Picard
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Scope and Sequence

Month	Pre-Calculus	Accelerated Pre-Calculus
September	**Prerequisites** -solve linear, quadratic, absolute value, square root equations -solve linear, quadratic, absolute value inequalities -graph linear, quadratic, absolute value, square root functions - simplify polynomial expressions -graph transformations Prerequisites Test **Functions and Their Graphs** - Represent relations using mappings, ordered pairs, tables, and graphs. - Determine whether a given relation is a function. - Identify the domain and range of any relation or function. - Use function notation and evaluate functions. - Identify and sketch the common families of functions: simple polynomial, square root, absolute value, step, piecewise, and rational. - Identify odd vs. even functions, as well as the related symmetry that exists in the graphs. - Determine and recognize inverse functions. - Use symmetry, vertical and horizontal stretches and shifts, and reflections to graph functions. - Perform operations with functions. - Find composites of functions.	**Prerequisites** -solve linear, quadratic, absolute value, square root equations -solve linear, quadratic, absolute value inequalities -graph linear, quadratic, absolute value, square root functions - simplify polynomial expressions -graph transformations Prerequisites Test **Functions and Their Graphs** - Represent relations using mappings, ordered pairs, tables, and graphs. - Determine whether a given relation is a function. - Identify the domain and range of any relation or function. - Use function notation and evaluate functions. - Identify and sketch the common families of functions: simple polynomial, square root, absolute value, step, piecewise, and rational. - Identify odd vs. even functions, as well as the related symmetry that exists in the graphs. - Determine and recognize inverse functions. - Use symmetry, vertical and horizontal stretches and shifts, and reflections to graph functions. - Perform operations with functions. - Find composites of functions. **Polynomial Functions** - Graph and analyze quadratic functions using equations in both quadratic and standard forms. - Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions. - Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function, as well as the number of critical points such as maxima, minima, points of inflection, and turning points. - Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. - Use factoring methods, The Rational Zero Theorem, long

		division, synthetic division, and The Remainder and Factor Theorems to identify rational zeros of a polynomial function. - Use all of the above, in addition to the Quadratic Formula and operations with complex numbers, to determine all zeros of a polynomial function, (real and complex), and to sketch its graph.
October	**Polynomial Functions** - Graph and analyze quadratic functions using equations in both quadratic and standard forms. - Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions. - Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function, as well as the number of critical points such as maxima, minima, points of inflection, and turning points. - Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. - Use factoring methods, The Rational Zero Theorem, long division, synthetic division, and The Remainder and Factor Theorems to identify rational zeros of a polynomial function. - Use all of the above, in addition to the Quadratic Formula and operations with complex numbers, to determine all zeros of a polynomial function, (real and complex), and to sketch its graph. **Rational Functions** - Determine horizontal, vertical, and slant asymptotes of rational functions, and use these to sketch the graphs, identify domains and ranges, and end behaviors. - Simplify rational expression - Solve rational equations - Decompose a rational expression into partial fractions.	**Rational Functions** - Determine horizontal, vertical, and slant asymptotes of rational functions, and use these to sketch the graphs, identify domains and ranges, and end behaviors. - Simplify rational expression - Solve rational equations - Decompose a rational expression into partial fractions. **Exponential and Logarithmic Functions** - Use the properties of exponents evaluate and simplify expressions containing rational and irrational exponents, and those that contain the natural base e. -Graph exponential functions and inequalities -Graph exponential functions and inequalities, including those that involve growth, decay, and the natural base e. -Evaluate and simplify logarithmic expressions. -Graph logarithmic functions. -Evaluate and graph natural logarithmic functions.
November	**Exponential and Logarithmic Functions** - Use the properties of exponents evaluate and simplify expressions containing rational and irrational exponents, and those that contain the natural base e. -Graph exponential functions and inequalities -Graph exponential functions and inequalities, including those that involve growth, decay, and the natural base e. -Evaluate and simplify logarithmic expressions. -Graph logarithmic functions. -Evaluate and graph natural logarithmic functions. -Use the properties of logarithms to evaluate, rewrite, expand, and condense logarithmic expressions. -Solve exponential and logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems.	**Exponential and Logarithmic Functions (Cont)** -Use the properties of logarithms to evaluate, rewrite, expand, and condense logarithmic expressions. -Solve exponential and logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems. **Trigonometric Functions of Angles** -Describe and measure angles using radian and degree measure. -Convert degree measure to radian measure and vice versa. -Evaluate trigonometric functions of any angle, given a point on its terminal side. -Evaluate trigonometric functions of any angle, with or without using a reference angle. -Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) -Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. -Identify trigonometric functions of special angles with or without the use of a unit circle. -Identify and construct a unit circle and recognize its relationship to real numbers. -Use a calculator to approximately evaluate a trigonometric function of any angle. -Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles. -Use the Law of Sines to solve oblique triangles. -Use the Law of Cosines to solve oblique triangles.

		-Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.
December	**Trigonometric Functions of Angles** -Describe and measure angles using radian and degree measure. -Convert degree measure to radian measure and vice versa. -Evaluate trigonometric functions of any angle, given a point on its terminal side. -Evaluate trigonometric functions of any angle, with or without using a reference angle. -Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) -Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. -Identify trigonometric functions of special angles with or without the use of a unit circle. -Identify and construct a unit circle and recognize its relationship to real numbers. -Use a calculator to approximately evaluate a trigonometric function of any angle. -Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles. -Use the Law of Sines to solve oblique triangles. -Use the Law of Cosines to solve oblique triangles. -Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.	**Trigonometric Functions of Real Numbers** - Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. - Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph. -Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts). - Evaluate, graph and identify the domains and ranges of inverse trigonometric functions. -Write equations for inverse trigonometric functions. - Evaluate compositions of trigonometric functions. - Use trigonometric functions and their inverses to model and solve real-life problems. - Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions. - Use the Fundamental Trigonometric Identities to verify other trigonometric identities. - Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions. - Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions. - Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.
January	**Trigonometric Functions of Real Numbers** - Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. - Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph. -Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts). - Evaluate, graph and identify the domains and ranges of inverse trigonometric functions. -Write equations for inverse trigonometric functions. - Evaluate compositions of trigonometric functions. - Use trigonometric functions and their inverses to model and solve real-life problems. - Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions. - Use the Fundamental Trigonometric Identities to verify other trigonometric identities. - Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions. - Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions. - Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.	**Vector** - Identify equal, opposite, and parallel vectors. - Add and subtract vectors geometrically. - Find ordered pairs that represent vectors. - Add, subtract, multiply, and calculate the magnitude of vectors algebraically. - Add and subtract and calculate the magnitude of vectors in three-dimensional space. - Calculate the inner and cross products of two vectors. - Determine whether two vectors are perpendicular. - Write vector and parametric equations of lines. - Graph parametric equations. - Use vectors and right triangle trigonometry to model and solve real-life problems. **Matrices and Systems of Equations** - Solve systems of equations by substitution, by elimination, by Gaussian elimination, and graphically. - Recognize linear systems in row-echelon form and to use back substitution to solve the system. - Solve nonsquare systems of equations. - Sketch the graphs of inequalities in two variables and solve systems of inequalities. - Solve linear programming problems. -Use systems of equations and inequalities to model and solve real life problems.
February	**Matrices and Systems of Equations**	**Matrices and Systems of Equations (Cont)** - Write matrices, identify their order, and perform elementary

	- Solve systems of equations by substitution, by elimination, by Gaussian elimination, and graphically. - Recognize linear systems in row-echelon form and to use back substitution to solve the system. - Solve nonsquare systems of equations. - Sketch the graphs of inequalities in two variables and solve systems of inequalities. - Solve linear programming problems. -Use systems of equations and inequalities to model and solve real life problems. - Write matrices, identify their order, and perform elementary row operations. - Add, subtract, and multiply two matrices, and multiply a matrix by a real number. -Verify that two matrices are inverses of each other and find inverses of matrices. - Use inverse matrices to solve systems of linear equations. - Find the determinants of square matrices. - Use Cramer's Rule to solve systems of linear equations.	row operations. - Add, subtract, and multiply two matrices, and multiply a matrix by a real number. -Verify that two matrices are inverses of each other and find inverses of matrices. - Use inverse matrices to solve systems of linear equations. - Find the determinants of square matrices. - Use Cramer's Rule to solve systems of linear equations.
March	**Conic Sections** - Graph and write equations of parabolas. - Graph and write equations of circles. - Graph and write equations of ellipses. - Calculate eccentricities of ellipses. - Graph and write equations of hyperbolas. - Classify a conic section using its general equation and/or its discriminant. - Graph and write equations and for transformed conic sections - Solve systems of quadratic systems algebraically and sketch the graph. - Use conics to model and solve real-life problems. **Vector** - Identify equal, opposite, and parallel vectors. - Add and subtract vectors geometrically. - Find ordered pairs that represent vectors. - Add, subtract, multiply, and calculate the magnitude of vectors algebraically. - Add and subtract and calculate the magnitude of vectors in three-dimensional space. - Calculate the inner and cross products of two vectors. - Determine whether two vectors are perpendicular. - Write vector and parametric equations of lines. - Graph parametric equations. - Use vectors and right triangle trigonometry to model and solve real-life problems.	**Sequences and Series** -Use sequence notation, factorial notation, and summation notation to represent sequences and series. - Write rules for arithmetic sequences that can be used to find any term in the sequence. - Find *n*th partial sums of an arithmetic series. - Write rules for geometric sequences and find sums of geometric series. - Find sums of infinite geometric series. - Use sequences and series to solve real-life problems.
April	**Polar Coordinates** - Plot points and find multiple representations of points in the polar coordinate system. - Convert points and equations from rectangular to polar form and vice versa. - Graph polar equations by point plotting, and also use symmetry, zeros, maximum r-values as graphing aides. - Recognize and sketch special polar graphs. **Sequences and Series** -Use sequence notation, factorial notation, and summation notation to represent sequences and series. - Write rules for arithmetic sequences that can be used to find any term in the sequence. - Find *n*th partial sums of an arithmetic series.	**Probability** - Use The Binomial Theorem to expand powers of binomials. -Use The Fundamental Counting Principle to solve counting problems. - Use permutations and combinations to solve counting problems. - Calculate probabilities of events: mutually exclusive and independent events, as well as complements of events. - Calculate the probability of an event using The Binomial Theorem **Conic Sections** - Graph and write equations of parabolas. - Graph and write equations of circles. - Graph and write equations of ellipses. - Calculate eccentricities of ellipses. - Graph and write equations of hyperbolas.

	- Write rules for geometric sequences and find sums of geometric series. - Find sums of infinite geometric series. - Use sequences and series to solve real-life problems.	- Classify a conic section using its general equation and/or its discriminant. - Graph and write equations and for transformed conic sections - Solve systems of quadratic systems algebraically and sketch the graph. - Use conics to model and solve real-life problems.
May	**Probability** - Use The Binomial Theorem to expand powers of binomials. -Use The Fundamental Counting Principle to solve counting problems. - Use permutations and combinations to solve counting problems. - Calculate probabilities of events: mutually exclusive and independent events, as well as complements of events. - Calculate the probability of an event using The Binomial Theorem	**Polar Coordinates** - Plot points and find multiple representations of points in the polar coordinate system. - Convert points and equations from rectangular to polar form and vice versa. - Graph polar equations by point plotting, and also use symmetry, zeros, maximum r-values as graphing aides. - Recognize and sketch special polar graphs. **Vectors** - Find the component form, the unit vector in the same direction, magnitude of vectors in space. - Find the dot product and angles between vectors in space. - Determine whether vectors in space are parallel or orthogonal. - Use vectors to solve real-life problems.
June	**Introduction to Calculus** -Use limit theorems to evaluate the limit of a polynomial function. -Explore techniques for evaluating limits: (direct substitution, dividing out, rationalizing, graphing) - Evaluate the limits of difference quotients. - Use a tangent line to approximate the slope of a graph at a point. - Use the definition of slope to find exact slopes of graph. - Find derivatives of polynomial functions. - Use derivatives to find slopes of graphs - Calculate derivatives using the Sum, Difference, Product, and Quotient Rules. - Calculate derivatives using The Chain Rule.	**Introduction to Calculus** -Use limit theorems to evaluate the limit of a polynomial function. -Explore techniques for evaluating limits: (direct substitution, dividing out, rationalizing, graphing) - Evaluate the limits of difference quotients. - Use a tangent line to approximate the slope of a graph at a point. - Use the definition of slope to find exact slopes of graph. - Find derivatives of polynomial functions. - Use derivatives to find slopes of graphs - Calculate derivatives using the Sum, Difference, Product, and Quotient Rules. - Calculate derivatives using The Chain Rule.

Unit 1

Families of Functions and their Graphs

Summary and Rationale

Unit 1 is a review and thorough extension of all families of functions and the characteristics of their graphs. Students will graph and analyze the behaviors of each family of functions, in order to develop a deeper understanding of how altering the equations of these functions affects all families in the same or very similar ways.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Algebra	
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude.
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.1	Write a functions that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard functions types using arithmetic operations. Compose functions
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.4	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or table, given that function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain.
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Given a two-variable equation, the value of one quantity can be found if the value of the other is known. A table can be used to display the relationship between the quantities, which would also represent a set of solutions of the equation. The set of all solutions of the equation forms its graph on a coordinate plane. The graph will show solutions that are in the table, will visually represent the relationship between the two variable quantities that are changing, and can also show solutions to the equation that are not in the table. A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena. Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range. Functions and their graphs can be classified into families of functions: linear (lines), quadratic (u-shaped), higher degree polynomial (also classified as cubic, even, or odd, shapes vary), square root (or radical), absolute value (v-shaped), greatest integer (or step), and rational. The graph of these functions in a coordinate plane can be created using a table of values or more quickly by shifting the following parent graphs: $y = x$ (linear) $y = x^2$ (quadratic) $y = x^3$, $y = x^4$, $y = x^5$ (polynomial)	What are the types and the differences between the following functions and how are the values of *a, h,* and *k* used to describe their graphs? $y = a\,(x - h) + k$ $y = a\,\lvert x - h \rvert + k$ $y = a\,(x - h)^2 + k$ $y = a\,(x - h)^n + k$

$y = \sqrt{x}$ (square root or radical),
$y = |x|$, (absolute value)
$y = [x]$ (greatest integer or step)
$y = 1/x$ (rational)

The graphs of functions in a coordinate plane vary, yet yield various patterns within the families of functions. Using algebraic methods to manipulate and/or solve the equation of a function can throw light on the function's properties such as zeros, intercepts, a vertex, direction of opening, end behavior, and type of symmetry, if it exists. These characteristics can help visualize the sketch of its graph and also lead to the use of more effective and efficient graphing methods.

Families of functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of *a, h,* and *k* similarly affect their graphs.

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line $y = x$.

Functions can be added, subtracted, multiplied, or divided to form a new function. They can also be combined or repeated to form a new function, called a composite function. The new functions created may have different domains and ranges than their parts.

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
- Families of functions - Graphing	- Represent relations using mappings, ordered pairs, tables, and graphs. - Determine whether a given relation is a function. - Identify the domain and range of any relation or function. - Use function notation and evaluate functions. - Identify and sketch the common families of functions: simple polynomial, square root, absolute value, step, piecewise, and rational. - Identify odd vs. even functions, as well as the related symmetry that exists in the graphs. - Determine and recognize inverse functions. - Use symmetry, vertical and horizontal stretches and shifts, and reflections to graph functions. - Perform operations with functions. - Find composites of functions.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student’s 504 plan. Examples include:

-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

LINKS TO CAREERS: https://www.math.uci.edu/math-majors/math-career-resources

Unit 2

Polynomial and Rational Functions

Summary and Rationale

The majority of Unit 2 is devoted to the study of polynomial functions, with a special focus on quadratic functions, which are degree two polynomials. Students will thoroughly analyze the characteristics and properties of these functions, their equations and graphs. Students will learn different solving and graphing methods, as well as how to write and obtain important information from function equations in different forms. During this unit, it is crucial for students to recognize that the *solutions* of an equation are equal to both the *zeros* of a function and the *x-intercept's* of this function's graph. This connection will follow through with other types of functions that will be studied in future units. The graph of a quadratic function is called a parabola, which is also one of the four conic sections that will be further explored in Unit 7.

The second part of the unit involves the further study of the rational function, which is the ratio of two polynomial functions. Students will analyze the characteristics of these functions, and their equations and graphs. Through this analysis, students will explore a family of functions that most often has domain and/or range restrictions as they draw graphs that contain asymptotes and other forms of discontinuity. Familiar concepts involving properties and operations of rational numbers will be extended to the process of decomposing rational expressions into partial fractions.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.CN.1	Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form a +b*i* with a and b real.
N.CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.
N.CN.8	Extend polynomial identities to the complex numbers
N.CN.9	Know the fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. m. Interpret parts of an expression, such as terms, factors, and coefficients. n. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expression for exponential functions.
A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the

	operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.2	Know and apply the remainder Theorem: For a polynomial p(x) and a number a, the remainder of division by x – a is p(a) = 0 if and only if (x – a) is a factor of p (x)
A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomials.
A.APR.4	Prove polynomial identities and use them to describe numerical relationships.
A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
A.APR.6	Rewrite simple ration expressions in different forms; write a(x) / b(x) in the form q(x) + r(x) / b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.7	Understand that rational expressions form a system of analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero ration expression; add, subtract, multiply, and divide rational expressions.
A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equations in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equations. Recognize when the quadratic formula gives complex solutions and write them as a + bi and a – bi for real numbers a and b.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).

F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.1	Write a functions that describes a relationship between two quantities. d. Determine an explicit expression, a recursive process, or steps for calculation from a context. e. Combine standard functions types using arithmetic operations. d. Compose functions
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.4	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or table, given that function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain.
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
All equations of degree one or higher are defined as polynomial equations. Familiar equations such as linear equations (degree 1) and quadratic equations (degree 2) are both examples of polynomial equations. The Fundamental Theorem of Algebra states that the number of solutions to a one-variable polynomial equation is equal to the degree of the polynomial. Quadratic equations are of degree two, therefore they have two solutions that can be determined from a graph (The solutions are the x-intercepts.), or by using different algebraic methods. Four previously learned algebraic methods are factoring, taking the square root of both sides, completing the square, or using the Quadratic Formula. The best method to use depends on the forms and characteristics of the given equations, the nature of the solutions, and how precise the solutions should be. Solutions to polynomial equations of degree greater than two may require the use of more algebraic methods and theorems in addition to those already mentioned such as synthetic or long division, The Remainder Theorem, Factor Theorem, Rational Root Theorem, Des Carte's Rule, and/or The Location Principle. Polynomial equations can have real number solutions, but can also have solutions in a larger system, called the complex numbers. There are differences between the results of operations on complex numbers from those obtained within the real number system. A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs	What is the reason for having the different methods for solving quadratics and when can the use of one method be necessary or more appropriate than another? What are the connections between the algebraic and graphical representations of both one-variable and two variable quadratic inequalities? What steps should be taken when factoring a polynomial expression or equation? What specifics do you look for and how do you know when it is completely factored? What characteristics, other than its solutions, of a polynomial equation can be used to describe the behavior and sketch the graph of its related function? What theorems and methods are used to identify the total number of roots and determine the real zeros of a polynomial function? Which types of roots are seen on the graph of a polynomial function? Which are not visible? What happens in the graph if there is a double root? How do you determine the vertical, horizontal, and/or slant asymptotes for the graph of a rational function? What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs? $y = a\,(x - h) + k$ $y = a\,\lvert x - h \rvert + k$ $y = a\,(x - h)^2 + k$ $y = a\,(x - h)^n + k$ $y = \frac{a}{(x - h)} + k$

represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena.

Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

The *solutions* of a polynomial equation are equal to the *real zeros* of the related polynomial function, and the *x-intercepts* of this function's graph. Zeros that are not real are not visible on the graph of a function in a coordinate plane of real numbers.

The graphs of polynomial functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a function can throw light on the function's properties such as: zeros, intercepts, a vertex, end behavior, direction of opening, domain, range, vertex, width, intervals of increasing or decreasing, relative maxima and/or minima, identification of double or triple roots, and other critical points. These characteristics can help visualize the sketch of its graph and can result in the use of more effective and efficient graphing methods.

Polynomial inequalities can be one variable or two, and both types can be solved and represented algebraically and graphically.

A rational function is the ratio of two polynomial functions. These functions contain restrictions on their domains and/or ranges. Therefore, their graphs contain asymptotes, holes, and/or discontinuity. The graphs of rational functions also vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a rational function can help determine important properties such as its zeroes, intercepts,

asymptotes, domain, range, types of discontinuity, and end behavior. These characteristics can help visualize the sketch of its graph and can lead the use of more effective and efficient graphing methods.

Polynomial and rational functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of *a, h,* and *k* similarly affect their graphs:

$y = a(x - h) + k$

$y = a(x - h)^2 + k$

$y = a(x - h)^n + k$

$y = \frac{a}{(x - h)} + k$

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
- Polynomials - Rational functions - Quadratic functions - Rational Zero Theorem - Graphing	- Graph and analyze quadratic functions using equations in both quadratic and standard forms. - Use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions. - Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function, as well as the number of critical points such as maxima, minima, points of inflection, and turning points. - Use graphing technology to identify the number of and approximate the real zeros of a polynomial function.- Use factoring methods, The Rational Zero Theorem, long division, synthetic division, and The Remainder and Factor Theorems to identify rational zeros of a polynomial function. - Use all of the above, in addition to the Quadratic Formula and operations with complex numbers, to determine all zeros of a polynomial function, (real and complex), and to sketch its graph.

	- Determine horizontal, vertical, and slant asymptotes of rational functions, and use these to sketch the graphs, identify domains and ranges, and end behaviors. - Decompose a rational expression into partial fractions.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

LINKS TO CAREERS: https://www.math.uci.edu/math-majors/math-career-resources

Unit 3

Exponential and Logarithmic Functions

Summary and Rationale

Unit 3 extends the study of functions to exponential and logarithmic functions, which are two important families of functions because they model many real-life situations. Students will analyze the characteristics and properties of these functions, their equations and graphs. Students will explore two more families of functions that contain asymptotes as they draw and recognize patterns within the behavior of exponential and logarithmic functions. Previously learned Properties of Exponents and new Properties of Logarithms will be used to simplify expressions and solve equations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity

N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. m. Interpret parts of an expression, such as terms, factors, and coefficients. n. Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expression for exponential functions.
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then f(x) denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

	a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.5	Understand that inverse relationships between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents
F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
Interdisciplinary Connections	
Standard x.x	
Integration of Technology	
Instructional Focus	

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena. Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range. A relationship between two variables or two sets of data is an exponential function if the two variables increase (grow) or decrease (decay) by the same *percent* over equal periods of time. The inverse of an exponential function of a logarithmic function of the same base. Natural exponential and logarithmic functions are of the natural base, *e*. Exponential functions have restrictions on the range therefore logarithmic functions contain domain restrictions. The graphs of these functions contain asymptotes, which are arbitrary lines that a graph approaches as you move away from the origin. The graphs of exponential and logarithmic functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a exponential or logarithmic function can throw light on the function's properties such as its zeroes and asymptotes, and can also help visualize the behavior of its graph. These strategies can result in the use of more effective and efficient graphing methods.	What values of b does $y = b^x$ represent exponential growth? Decay? How do you determine the asymptotes of an exponential or logarithmic graph? What are the types and the differences between the two functions and how are the values of *a, h,* and *k* used to describe their graphs? $y = a(x - h) + k$ $y = a\lvert x - h\rvert + k$ $y = a(x - h)^2 + k$ $y = a(x - h)^n + k$ $y = \frac{a}{(x - h)} + k$ $y = ab^{(x-h)} + k$ $y = \log_b(x - h) + k$ How can you use inverse functions to graph logarithmic functions? How is solving a logarithmic equation similar to solving an exponential equation? How is it different? Why do logarithmic equations sometimes contain extraneous solutions?

Families of functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of *a, h,* and *k* similarly affect their graphs:

§ $y = a(x - h) + k$
§ $y = a|x - h| + k$
§ $y = a(x - h)^2 + k$
§ $y = a(x - h)^n + k$
§ $y = \frac{a}{(x - h)} + k$
§ $y = ab^{(x-h)} + k$
§ $y = \log_b(x - h) + k$

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line y = x.

The use of inverse operations with the properties of equality, exponents, and logarithms can be applied to the solving of exponential and logarithmic equations. In both cases, it is useful to isolate the exponential or logarithmic first, and then use inverse operations to solve for the variable. Since the domains of logarithmic functions contain restrictions, obtaining extraneous solutions is a possibility.

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Exponential functions · Inverse functions · Logarithmic equations	·Use the properties of exponents evaluate and simplify expressions containing rational and irrational exponents, and those that contain the natural base *e*. ·Graph exponential functions and inequalities ·Graph exponential functions and inequalities, including those that involve growth, decay, and the natural base *e*. ·Evaluate and simplify logarithmic expressions. ·Graph logarithmic functions. ·Evaluate and graph natural logarithmic functions. ·Use the properties of logarithms to evaluate, rewrite, expand, and condense logarithmic expressions. ·Solve exponential and logarithmic equations. ·Use exponential and logarithmic functions to model and solve real-life problems.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student’s 504 plan. Examples include:

-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
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- ❑ Use technology to enhance productivity.

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Unit 4

Trigonometric Functions of Angles

Summary and Rationale

Unit 4 continues the study of trigonometry, with a specific focus on the study of angles and triangles. Trigonometry is closely tied to both algebra and geometry. Students began their study of trigonometry in their geometry courses with right triangles. This unit will provide opportunities for students to further explore the more complex and comprehensive ideas behind the use of trigonometry in triangle measurement. Students will find trigonometric functions of angles, solve triangles, and use trigonometry to calculate the area of triangles.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Functions	
F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4, and p/6 and use the unit circle to express the values of sine, cosine, and tangent for p - x, p + x and 2p - x in terms of their values for x, where x is any real number.
F.TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
Geometry	
G.SRT.7	Explain and use the relationship between the side and cosine complementary angles
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.9	Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10	Prove the Laws of Sines and Cosines and use them to solve problems
G.SRT.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Interdisciplinary Connections	
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Trigonometric functions of acute angles are defined by ratios of side lengths of right triangles. These ratios are: sine, cosine, tangent, cosecant, secant, and cotangent.	How do you convert between degrees and radians? How can you use the trigonometric ratios to find the measurements of sides or angles of a right triangle?

The angles 30°, 45°, and 60° occur frequently in trigonometry. These are the angles of the two special right triangles previously learned in geometry (30°-60°-90° and 45°-45°-90°) The values of the six trigonometric functions of these angles do not need to be memorized since they can quickly be generated by drawing the two special right triangles and labeling the sides of each.

Trigonometric functions and their inverses can be used to find a missing side length or angle measure in a right triangle. Finding all missing lengths and angle measures is called solving a right triangle.

An angle is formed by two rays that have a common endpoint, called the vertex. You can generate an angle by fixing one ray, called the initial side, and rotating the other ray, called the terminal side, about the vertex. In a coordinate plane, an angle whose vertex is at the origin and whose initial side is the positive x-axis on in standard position. The phrase "the terminal side of θ lies in quadrant…" is simply saying that θ lies in that quadrant. Two angles in standard position are co-terminal if their terminal sides coincide. An angle co-terminal with a given angle can be found by adding or subtracting multiples of 360°.

Angles can be measured in degrees and in radians. The measure of an angle is determined by the amount and direction of a rotation from the initial side to the terminal side. The angles measure is positive is the rotation is counterclockwise and negative if the rotation is clockwise. The terminal side of an angle can make more than one complete rotation.

To define a radian, consider a circle with radius r centered at the origin. One radian is the measure of an angle in standard position whose terminal side intercepts the arc of length r.
Since the circumference of a circle is equal to $2\pi r$, there are 2π radians in a circle. Therefore 2π radians is equivalent to 360°. (π radians = 180°, and so on…)
The length of an arc and the area of a sector of a circle can be calculated using the central angle of the circle in degrees (previously learned), and in radians.

Reference angles are used to evaluate trigonometric functions of *any* angle.

How can you use The Law of Sines and The Law of Cosines to solve any triangle?

The Law of Sines and The Law of Cosines can be used to solve *any* triangle. (Determine all missing side lengths of angle measurements.) The use of one law rather than the other depends on the known information about he triangle. Some problems may show cases in which a triangle cannot be made under the given conditions (No solution). Other cases yield one or two possible triangle solutions. Trigonometric ratios can also be used to calculate the area of *any* triangle.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know: · Trigonometric functions · Law of Sines · Law of Cosines · Heron's Area Formula	Students will be able to: ·Describe and measure angles using radian and degree measure. ·Convert degree measure to radian measure and vice versa. ·Evaluate trigonometric functions of any angle, given a point on its terminal side. ·Evaluate trigonometric functions of any angle, with or without using a reference angle. ·Identify trigonometric functions of any angle using the period of the function, (or by identifying and using co-terminal angles.) ·Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions. ·Identify trigonometric functions of special angles with or without the use of a unit circle. ·Identify and construct a unit circle and recognize its relationship to real numbers. ·Use a calculator to approximately evaluate a trigonometric function of any angle. ·Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles. · Use the Law of Sines to solve oblique triangles. · Use the Law of Cosines to solve oblique triangles.

	·Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle. .

Suggested Resources/Technology Tools

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-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

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-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

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- ❑ Apply appropriate academic and technical skills.
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- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
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Unit 5

Trigonometric Functions of Real Numbers

Summary and Rationale

The focus of Unit 5 is the study of trigonometric functions of real numbers, used to model periodic behavior. Students will have opportunities to see more connections between trigonometry and algebra as they graph trigonometric functions and their inverses in a coordinate plane. Just as they have done with other types of functions in previous units, students will analyze the characteristics of their equations and graphs. Students will also use advanced algebra skills to evaluate trigonometric functions, verify trigonometric identities, and solve trigonometric equations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Algebra	
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Functions	
F.IF.4	For a function that models a relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showering period, midline, and amplitude.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F.TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.8	Prove the Pythagorean identity $\sin^2(q) + \cos^2(q) = 1$ and use it to find sin(q), cos(q), or tan(q) given sin(q), cos(q), or tan(q) and the quadrant of the angle.
F.TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	
G.SRT.7	Explain and use the relationship between the side and cosine complementary angles.
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena. Periodic functions have graphs that have repeating patterns that continue indefinitely. The shortest repeating portion is called a cycle. The horizontal length of each cycle is called the period. Trigonometric functions are periodic functions. Determining an output value of a trigonometric function, given an input value, requires evaluating the trigonometric expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range. The graphs of the sine and cosine are periodic functions that both have the following characteristics: The domain of each function is all real numbers and the range of each function is $-1 \leq y \leq 1$. Each graph has a period of 2π (or 360°.) The graph of tangent is a periodic function with a domain of all real numbers except odd multiples of $\pi/2$. At odd multiples of $\pi/2$, the graph has vertical asymptotes. The	Why are trigonometric functions called periodic functions? What is a trigonometric identity? How can trigonometric identities be used to determine exact trigonometric values of uncommon angles? How can you solve trigonometric equations?

range is all real numbers. The graph has a period of π (or 180°.)

Families of functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of *a, h,* and *k* similarly affect their graphs:

$y = a(x - h) + k$

$y = a|x - h| + k$

$y = a(x - h)^2 + k$

$y = a(x - h)^n + k$

$y = \frac{a}{(x - h)} + k$

$y = ab^{(x-h)} + k$

$y = \log_b(x - h) + k$

$y = a \sin b(x - h) + k$

$y = a \cos b(x - h) + k$

$y = a \tan b(x - h) + k$

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line y = x.

Trigonometric and inverse trigonometric functions and their graphs can be used to model and solve real-world problems, particularly those that involve periodic behavior.

Trigonometric identities are equations that are true for all values of x in their domain. Examples of Fundamental Trigonometric Identities are: Reciprocal Identities, Tangent and Cotangent Identities, Pythagorean Identities, Co-function Identities, and Negative Angle Identities. These trigonometric identities can be used to evaluate trigonometric functions, simplify trigonometric expressions, and verify other identities.

A verification of a trigonometric identity is a chain of equivalent expressions showing that one side of the identity is equal to the other side. When verifying an identity a common and useful strategy is to begin with the expression from one side and manipulate it algebraically until it is identical to the other side.

Properties of real numbers and equality, along with the use of inverse operations and trigonometric identities, can

transform a trigonometric equation into one or a series of equivalent simpler equations. These properties can be used repeatedly to isolate the variable. Algebraic techniques such as factoring or The Quadratic Formula may also be necessary during the process of finding solutions to trigonometric equations in quadratic form. Extraneous solutions can result if an equation is being solved within a restricted domain. Sum, Difference, Double, and Half-Angle Formulas make it possible to determine exact trigonometric values for less common angles using the more common angles such as 30°, 45°, 60°, etc.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know: · Trigonometric functions of real numbers · Formulas · Graphing	Students will be able to: ·Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant. ·Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph. ·Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts). · Evaluate, graph and identify the domains and ranges of inverse trigonometric functions. · Write equations for inverse trigonometric functions. · Evaluate compositions of trigonometric functions. · Use trigonometric functions and their inverses to model and solve real-life problems. · Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions. · Use the Fundamental Trigonometric Identities to verify other trigonometric identities. · Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions.

	· Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions. · Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

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-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

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Unit 6

Vectors

Summary and Rationale

Unit 6 discusses vectors. Vectors are an important topic in mathematics because of their application in calculus and physics.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.VM.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes.
N.VM.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
N.VM.3	Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. Understand vector subtraction v – w as v + (-w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise
N.VM.5	Multiply vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g. as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cv using $\|cv\| = \|c\|v$. Compute the direction of cv knowing that when $\|c\|v$ ¹ 0, the direction of cv is either along v or against v.
Interdisciplinary Connections	
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
Vectors are used to mathematically represent velocities. A vector is a quantity, or directed distance, that ahs both magnitude and direction. A vector is represented geometrically by a directed line segment. The length of the directed line segment is the vector's magnitude. If a vector has an initial point at the origin, it is in standard position. The amplitude of the vector is the directed angle between the positive x-axis and the vector. The sum of two or more vectors is called the resultant of the vectors. The resultant of vectors can be found through two methods: The Parallelogram Method and The Triangle Method (also called The Tip-To Tail Method.)	What is a vector and why are vectors used? How can vectors be represented? How do you add vectors? Subtract?

Two vectors are opposites of they have the same magnitude and opposite directions. Two vectors are parallel if and only if they have the same or opposite directions. Two or more vectors whose sum is a given vector are called components of the given vector. Components can have any direction. Vectors are used in physics to represent motion or forces acting upon objects. Vectors can be represented algebraically using ordered pairs of real numbers. Since vectors with the same magnitude and amplitude are equal, many vectors can be represented by an ordered pair.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know: · Vectors · Parametric equations	Students will be able to: · Identify equal, opposite, and parallel vectors. · Add and subtract vectors geometrically. · Find ordered pairs that represent vectors. · Add, subtract, multiply, and calculate the magnitude of vectors algebraically. · Add and subtract and calculate the magnitude of vectors in three-dimensional space. · Calculate the inner and cross products of two vectors. · Determine whether two vectors are perpendicular. · Write vector and parametric equations of lines. · Graph parametric equations. · Use vectors and right triangle trigonometry to model and solve real-life problems.

Suggested Resources/Technology Tools

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Unit 7

Polar Coordinates and Conic Sections

Summary and Rationale

This unit will introduce students to graphing on a new set of axes, the polar axes. Unit 7 involves the study of the four conic sections, parabolas, circles, ellipses, and hyperbolas. Conic sections are an important part of the study of algebra and geometry because they have many different real-life applications. This unit also introduces the polar coordinate system which provides us with another method for describing locations, in addition to the well-known coordinate plane system.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.Q.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
N.Q.6	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
Algebra	
A.SSE.1	Interpret expressions that represent a quantity in terms of its context. a) Interpret parts of an expression, such as terms, factors, and coefficients.

	b) Interpret complication expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a) Factor a quadratic expression to reveal the zeros of the function it defines. b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c) Use the properties of exponents to transform expression for exponential functions
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Geometry	
G.GPE.1	Derive the equations of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and the radius of a circle given by an example.
G.GPE.2	Derive the equation of a parabola given a focus and directrix
G.GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of the distances from the foci is constant
Interdisciplinary Connections	
Standard x.x	
Integration of Technology	
Instructional Focus	

Enduring Understandings:	**Essential Questions:**
The Distance Formula is an application of the previously learned Pythagorean Theorem. A conic section is a graph of an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ These relations are studied in algebra because they have various real-life applications. The only conic sections that are functions are parabolas that open upward or downward, previously learned as quadratic functions and hyperbolas that are written in the form of a rational function. Using algebra to manipulate the equation of a conic section, particularly the method of "completing the square" can be used to determine the parts and properties of its graph, and can result in the use of more effective and efficient graphing methods. Two quadratic equations form a quadratic system. The solutions of this type of system are represented by the set of ordered pairs that satisfy both equations in the system. The solutions of these systems can be determined and represented both algebraically and graphically, using the same algebraic methods previously used to solve linear systems. However, solving quadratic systems requires methods learned for solving quadratic equations, and may yield more solutions.	Given the equation of a conic section in the following form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ how do you classify its graph? How do you rewrite the equation so that you can obtain the information needed to draw its graph? Which conic sections are functions? Which are relations, but not functions? How many solutions are possible for a system containing a conic section and a line? For a system of two conic sections? What would each possible solution case look like graphically? What algebraic methods can be used to solve a system of equations that contains one or more quadratic relation? How is the polar coordinate system different from the four-quadrant coordinate plane system?
Evidence of Learning (Assessments)	
Tests Quizzes Homework Class participation	
Objectives (SLO)	

Students will know: · Polar coordinates · Conic sections · Graphing	Students will be able to: · Graph and write equations of parabolas. ·Graph and write equations of circles. ·Graph and write equations of ellipses. ·Calculate eccentricities of ellipses. ·Graph and write equations of hyperbolas. ·Classify a conic section using its general equation and/or its discriminant. ·Graph and write equations and for transformed conic sections ·Solve systems of quadratic systems algebraically and sketch the graph. ·Use conics to model and solve real-life problems. · Plot points and find multiple representations of points in the polar coordinate system. ·Convert points and equations from rectangular to polar form and vice versa. ·Graph polar equations by point plotting, and also use symmetry, zeros, maximum r-values as graphing aides. ·Recognize and sketch special polar graphs.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
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Modifications

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Unit 8

Matrices and Systems of equations

Summary and Rationale

Unit 8 involves the study of matrices, which are used throughout linear algebra to solve linear systems. This unit will provide an introduction to matrices, their uses, and the properties of matrix operations.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards	
Number and Quantity	
N-VM-6	With prompting and support, ask and answer questions about key details in a text.
N-VM-7	Multiply matrices by scalars to produce new matrices, e.g., as when all the payoffs in a game are doubled.
N-VM-8	Add, subtract, and multiply matrices of appropriate dimensions.
N-VM-9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
N-VM-10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
N-VM-11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
N-VM-12	Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
Algebra	
A-REI-8	Represent a system of linear equations as a single matrix equation in a vector variable.
A-REI-9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).
Interdisciplinary Connections	
Integration of Technology	

Instructional Focus	
Enduring Understandings:	**Essential Questions:**

Solving an equation (or inequality) is the process of rewriting the equation (or inequality) to make what it says about its variable(s) as simple as possible. Properties of numbers, equality, and inequality can be used to transform an equation (or inequality) into equivalent, simpler equations (or inequalities) in order to find solutions. Two or more equations/inequalities form a system. The solution of a system is represented by the set of ordered pairs that satisfy every equation/inequality in the system. Systems of equations can be solved in more than one way. Three methods are graphing, substitution, and elimination. The best method to use depends on the forms of the given equations and how precise the solution should be. The graphing method involves graphing each equation and finding the intersection point, if one exists. When a system has at least one equation that can be solved for a variable, the system can be efficiently solved using substitution. Some equations of a system are written in a way that makes eliminating a variable the best method to use. A matrix is a rectangular arrangement of numbers in rows and columns that can be used to organize and perform operations on numerical data. The numbers in a matrix are called its entries, (or elements.) To add or subtract matrices, add or subtract corresponding entries. Matrices can be added or subtracted only if they have the same dimensions. The entries in a matrix can also be multiplied by a real number. This process is called scalar multiplication. The product of two matrices A and B is defined if the number of columns in A is equal to the number of rows in B. Some of the properties of operations of real numbers differ from the properties of matrix operations. One major difference is as follows: Matrix multiplication, in general is not commutative. Square matrices (matrices with the same number of rows and columns) have multiplicative inverses, as	How can you use the graph and/or the equations of a linear system to determine the number of solutions, as well as the actual solution? When and why is using one method for solving a linear system sometimes more appropriate than another? What are the connections between the algebraic and graphical representations of a system of linear inequalities?

long as their determinants do not equal zero. There are formulas and procedures for determining the inverse of a square matrix that involve the determinant of the matrix. Matrices, their determinants and inverses, and matrix operations can be used to solve linear systems through the application of Cramer's Rule or a matrix equation.	

Evidence of Learning (Assessments)

Objectives (SLO)

Students will know: ·Systems ·Solving equations with different numbers of variables ·Graphing ·Matrices ·Determinants ·Cramer's Rule	Students will be able to: · Graph and solve systems of linear equations in two-variables. · Solve linear systems using algebraic methods. · Graph a system of linear inequalities to determine and represent the solutions of the system. · Solve linear programming problems. · Solve systems of linear equations in three variables. · Use linear systems and linear programming to model and solve real-life problems. · Add and subtract matrices and multiply a matrix by a scalar. · Solve matrix equations. · Multiply two matrices. · Evaluate determinants of 2x2 and 3x3 matrices. · Find the inverse of a matrix. · Solve linear systems using Cramer's Rule. · Solve linear systems using the inverse of a matrix.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
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Modifications

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Unit 8

Sequences, Series and Probability

Summary and Rationale

Students have studied number patterns (or sequences) since elementary school, and again in Algebra. Unit 8 will connect and extend this familiar exploration of sequences and series to calculation probabilities and binomial expansion.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Algebra	
A-SSE-4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
Functions	
F-IF-3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
F-BF-2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between two forms.
Interdisciplinary Connections	
Integration of Technology	

Instructional Focus	
Enduring Understandings:	**Essential Questions:**
Algebra can be used to write and use rules for number patterns that exist in sequences and series. A sequence is a list of terms that demonstrate a number pattern, while a series is an expression formed by adding the terms of the sequence. Both can be either finite or infinite. An arithmetic sequence has a common difference, so it is similar to a linear function. A geometric sequence has a common ratio, so it is similar to an exponential function. A recursive sequence is one that uses one or more pervious term(s) to obtain the next term. Probability expresses the likelihood that a particular event will occur. Data can be used to calculate experimental probability, and mathematical properties can be used to calculate theoretical probability. Either experimental or theoretical probability can be used to make predictions or decisions about future events. The probability of an event, or P(event), tells how likely it is that the event will occur. The probability of a compound event can sometimes be found from expressions of the probabilities of simpler events. Different methods must be used for finding the probability of two dependent events compared to finding the probability of two independent events. Various counting methods can be used to find the number of possible ways to choose objects with and without order, which can be further used to develop theoretical probabilities. Permutation and combination notation can be used to represent real-world situations. The Binomial Theorem can be used to calculate the probability of a real-world event and can also be used to identify the terms of a binomial expansion.	What is the difference between a sequence and a series? What makes a sequence or series arithmetic? geometric? What information do you need to find a term of an arithmetic (or geometric) series? How do you know if a sum of an infinite geometric series exists? And if it does, what information is needed to determine the sum? How can summation (or sigma) notation be used to represent a series? How is probability related to real-world events? How can you use the binomial theorem to calculate the probability of a real-world event?

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know:	Students will be able to:
· Sequence · Series · Probability	· Use sequence notation, factorial notation, and summation notation to represent sequences and series. · Write rules for arithmetic sequences that can be used to find any term in the sequence. · Find *n*th partial sums of an arithmetic series. · Write rules for geometric sequences and find sums of geometric series. · Find sums of infinite geometric series. · Use sequences and series to solve real-life problems. · Use The Binomial Theorem to expand powers of binomials. · Use The Fundamental Counting Principle to solve counting problems. · Use permutations and combinations to solve counting problems. · Calculate probabilities of events: mutually exclusive and independent events, as well as complements of events. · Calculate the probability of an event using The Binomial Theorem

Suggested Resources/Technology Tools

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Unit 9

Introduction to Calculus

Summary and Rationale

Unit 10 is an introduction to the most important concepts and ideas involved in the study of Calculus. The introduction of differential calculus will be explored as students learn how to find the limit and derivative of a function.

Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

Standards

Number and Quantity	
N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
Algebra	
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations**.** Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for p/3, p/4, and p/6 and use the unit circle to express the values of sine, cosine, and tangent for p - x, p + x and 2p - x in terms of their values for x, where x is any real number.
F.TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always

	decreasing allows its inverse to be constructed.
F.TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.8	Prove the Pythagorean identity $\sin^2(q) + \cos^2(q) = 1$ and use it to find sin(q), cos(q), or tan(q) given sin(q), cos(q), or tan(q) and the quadrant of the angle.
F.TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.4	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or table, given that function has an inverse. Produce an invertible function from a non-invertible function by restricting the domain.
Interdisciplinary Connections	
Integration of Technology	

Instructional Focus

Enduring Understandings:	**Essential Questions:**
A function is a relationship between variables in which each value of the input variable (domain) is associated with a unique value of the output variable (range). Functions can be represented in a variety of ways, such as graph, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function. Limits provide us with language for describing how the outputs of a function behave as the inputs approach some particular value.	What does the derivative of a function represent? How can you find the derivative of a function using the definition? How can you find the derivative of a function using the Product Rule? Quotient Rule? Chain Rule?

Limits can also be determined as the inputs approach positive or negative infinity. In these cases, limits can be used to describe the end behavior of a function and its graph. The Properties of Limits can be used to calculate unfamiliar limits by using limits that we already know. Sometimes the values of a function tend to different limits as x approaches a number c from opposite sides. In this case, these are the right-hand limit and a left-hand limit. The slope of a tangent to a curve can be found by calculating the limit of the slopes of secant lines. (using the difference quotient). The slope of a tangent to a curve can also be calculated by finding the derivative of the function through its definition or using one or more of the Rules for Differentiation.	

Evidence of Learning (Assessments)

Tests
Quizzes
Homework
Class participation

Objectives (SLO)

Students will know: · Limits · Derivatives · Slope of a Curve	Students will be able to: · Use limit theorems to evaluate the limit of a polynomial function. · Explore techniques for evaluating limits: (direct substitution, dividing out, rationalizing, graphing) · Evaluate the limits of difference quotients. · Use a tangent line to approximate the slope of a graph at a point. · Use the definition of slope to find exact slopes of graph. · Find derivatives of polynomial functions. · Use derivatives to find slopes of graphs · Calculate derivatives using the Sum, Difference, Product, and Quotient Rules.

	· Calculate derivatives using The Chain Rule.

Suggested Resources/Technology Tools

-Textbooks, workbooks, and assessment aides
-Online textbook
-Parcc.pearson.com & parcconline.org (PARCC Practice Tests and Released Items)
-Khan Academy; www.insidemathematics.org/performanceassessment-tasks
-Calculators when specified
-Google Classroom
-Desmos graphing calculator
-Kuta Software

Modifications

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:
-Use concrete examples of concepts before teaching the abstract
-Reduce the number of concepts presented at one time
-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
-Use of aids (calculator, computer, tape recorder, etc.)
-Frequently check on progress of independent work
-Provide study guides and copy of notes
-Provide repetition and practice

ELL: Modifications are determined by each student. Examples include:
- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

504: Modifications are determined by each student's 504 plan. Examples include:
-Teacher will review, restate and repeat directions, as needed
-Frequently check on progress of independent work

21ST CENTURY LIFE AND CAREER STANDARDS

- ❑ Apply appropriate academic and technical skills.
- ❑ Communicate clearly and effectively and with reason.
- ❑ Demonstrate creativity and innovation.
- ❑ Employ valid and reliable research strategies.
- ❑ Utilize critical thinking to make sense of problems and persevere in solving them.
- ❑ Model integrity, ethical leadership, and effective management.
- ❑ Plan education and career paths aligned to personal goals.
- ❑ Use technology to enhance productivity.

LINKS TO CAREERS: https://www.math.uci.edu/math-majors/math-career-resources