



**Mathematics Department**  
Algebra 2

**Developed by:** Leann Martin and Jessica Mabel

**Effective Date:** September 2023

**Scope and Sequence**

Month	Algebra 2	Advanced Algebra 2	Accelerated Algebra 2
September	<p><b>Extensive Review of Algebra 1 Prerequisite Skills:</b></p> <ul style="list-style-type: none"> <li>-Linear equations and inequalities</li> <li>-System of linear equations and inequalities</li> <li>- Order or Operations</li> <li>-Quadratics- Factoring</li> </ul> <p><b>(See Algebra 1)</b></p>	<p><b>Review of Algebra 1 Prerequisite Skills:</b></p> <ul style="list-style-type: none"> <li>-Linear equations and inequalities</li> <li>-System of Linear equations and inequalities</li> <li>- Order or Operations</li> <li>-Quadratics- Factoring</li> </ul> <p><b>Prerequisites Assessment</b></p>	<p><b>Review of Algebra 1 Prerequisites</b></p> <ul style="list-style-type: none"> <li>-Linear equations and inequalities</li> <li>-System of equations and inequalities</li> <li>- Order or Operations</li> <li>-Quadratic Equations</li> <li>-Functions</li> </ul> <p>Prerequisites skills will be reviewed through independent study and making use of teacher's office hours</p> <p><b>Prerequisites Assessment</b></p> <p><b>Unit 1: Quadratic Functions</b></p> <ul style="list-style-type: none"> <li>- Graph quadratic functions.</li> <li>- Factor quadratic expressions and solve quadratic equations by factoring.</li> <li>- Find the zeros of a quadratic function by factoring.</li> </ul>

			<ul style="list-style-type: none"> <li>- Solve quadratic equations by taking the square root of both sides.</li> <li>- Perform operations on complex numbers.</li> <li>- Solve quadratic equations by completing the square.</li> <li>- Use completing the square to write quadratic functions in vertex form <math>y = a(x - h)^2 + k</math>.</li> <li>- Solve quadratic equations using the quadratic formula.</li> <li>- Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving.</li> <li>- Write and graph quadratic functions in standard form (<math>y = ax^2 + bx + c</math>), vertex form (<math>y = a(x - h)^2 + k</math>), and intercept form (<math>y = a(x - p)(x - q)</math>).</li> <li>- Solve quadratic inequalities in one variable by graphing and algebraically.</li> <li>-Graph quadratic inequalities in two variables to show all solutions.</li> <li>-Use quadratic equations, inequalities, and functions to model and solve real-life problems.</li> </ul>
<p><b>October</b></p>	<p><b>Extensive Review of Prerequisite Skills (Cont)</b> (See Algebra 1)</p> <p><b>Prerequisites Assessment</b></p>	<p><b>Unit 1: Quadratic Functions</b></p> <ul style="list-style-type: none"> <li>- Graph quadratic functions.</li> <li>- Factor quadratic expressions and solve quadratic equations by factoring.</li> </ul>	<p><b>Unit 2: Polynomial Functions</b></p> <ul style="list-style-type: none"> <li>-Apply the properties of exponents to simplify and evaluate expressions involving powers.</li> <li>-Evaluate a polynomial function.</li> </ul>

	<p><b>Begin Unit 1: Quadratic Functions</b></p> <ul style="list-style-type: none"> <li>- Graph quadratic functions.</li> <li>- Factor quadratic expressions and solve quadratic equations by factoring.</li> </ul>	<ul style="list-style-type: none"> <li>- Find the zeros of a quadratic function by factoring.</li> <li>- Solve quadratic equations by taking the square root of both sides.</li> <li>- Perform operations on complex numbers.</li> <li>- Solve quadratic equations by completing the square.</li> <li>- Use completing the square to write quadratic functions in vertex form <math>y = a(x - h)^2 + k</math>.</li> <li>- Solve quadratic equations using the quadratic formula.</li> <li>- Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving.</li> <li>- Write and graph quadratic functions in standard form (<math>y = ax^2 + bx + c</math>), vertex form (<math>y = a(x - h)^2 + k</math>), and intercept form (<math>y = a(x - p)(x - q)</math>).</li> <li>- Solve quadratic inequalities in one variable by graphing and algebraically.</li> <li>-Graph quadratic inequalities in two variables to show all solutions.</li> <li>-Use quadratic equations, inequalities, and functions to model and solve real-life problems.</li> </ul>	<ul style="list-style-type: none"> <li>-Graph a polynomial function.</li> <li>-Add, subtract, and multiply polynomials.</li> <li>-Factor polynomial expressions.</li> <li>-Use factoring methods to solve polynomial equations.</li> <li>-Divide polynomials using long and synthetic division.</li> <li>-Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.</li> <li>-Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function.</li> <li>-Use graphing technology to identify the number of and approximate the real zeros of a polynomial function.</li> <li>-Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function.</li> <li>-Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph.</li> <li>-Use polynomial equations and functions to model and solve real-life problems.</li> </ul>
November	<p><b>Unit 1: Quadratic Functions (Cont)</b></p> <p>Find the zeros of a quadratic function by factoring.</p>	<p><b>Unit 1: Quadratic Functions (Cont)</b></p> <ul style="list-style-type: none"> <li>- Solve quadratic</li> </ul>	<p><b>Unit 3: Powers and Roots</b></p> <ul style="list-style-type: none"> <li>-Evaluate <math>n</math>th roots of real numbers using both radical and rational exponent</li> </ul>

	<ul style="list-style-type: none"> <li>- Solve quadratic equations by taking the square root of both sides.</li> <li>- Perform operations on complex numbers.</li> <li>- Solve quadratic equations by completing the square.</li> <li>- Use completing the square to write quadratic functions in vertex form <math>y = a(x - h)^2 + k</math>.</li> <li>-</li> </ul>	<p>inequalities in one variable by graphing and algebraically.</p> <ul style="list-style-type: none"> <li>-Graph quadratic inequalities in two variables to show all solutions.</li> <li>-Use quadratic equations, inequalities, and functions to model and solve real-life problems.</li> </ul>	<p>notation.</p> <ul style="list-style-type: none"> <li>-Apply the properties of exponents to simplify and evaluate expressions containing rational exponents.</li> <li>-Perform operations with functions, including those that contain powers.</li> <li>-Find the inverse of both linear and nonlinear functions.</li> <li>-Graph square root and cube root functions.</li> <li>-Solve equations that contain radicals and rational exponents.</li> <li>-Use radical and <math>n</math>th root equations and functions to model and solve real-life problems.</li> </ul>
<p><b>December</b></p>	<p><b>Unit 1: Quadratic Functions (Cont)</b> Solve quadratic equations using the quadratic formula.</p> <ul style="list-style-type: none"> <li>- Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving.</li> </ul>	<p><b>Unit 2: Polynomial Functions</b></p> <ul style="list-style-type: none"> <li>-Apply the properties of exponents to simplify and evaluate expressions involving powers.</li> <li>-Evaluate a polynomial function.</li> <li>-Graph a polynomial function.</li> <li>-Add, subtract, and multiply polynomials.</li> <li>-Factor polynomial expressions.</li> <li>-Use factoring methods to solve polynomial equations.</li> <li>-Divide polynomials using long and synthetic division.</li> <li>-Relate the results of</li> </ul>	<p><b>Unit 4: Exponential and Logarithmic Functions</b></p> <ul style="list-style-type: none"> <li>-Graph exponential functions, including those that model growth and decay.</li> <li>-Graph exponential functions that contain the natural base <math>e</math>.</li> <li>-Evaluate and simplify exponential expressions with base <math>e</math>.</li> <li>-Evaluate and simplify logarithmic expressions.</li> <li>-Graph logarithmic functions.</li> <li>-Use the properties of logarithms.</li> <li>-Solve exponential and</li> </ul>

		division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.	logarithmic equations. -Use exponential and logarithmic functions to model and solve real-life problems.
<b>January</b>	<b>Unit 2: Polynomial Functions</b> -Apply the properties of exponents to simplify and evaluate expressions involving powers. -Add, subtract, and multiply polynomials. -Factor polynomial expressions. -Use factoring methods to solve polynomial equations. -Divide polynomials using long and synthetic division. -Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.	<b>Unit 2: Polynomial Functions (Cont)</b> -Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. -Use graphing technology to identify the number of and approximate the real zeros of a polynomial function. -Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function. -Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph. -Use polynomial equations and functions to model and solve real-life problems.	<b>Unit 5: Rational Functions</b> -Graph rational functions. -Write and use equations for inverse and joint variations. -Multiply and divide rational expressions. -Add and subtract rational expressions. -Simplify complex fractions. -Solve rational equations. Use rational equations and functions to model and solve real-life problems.
<b>February</b>	<b>Unit 2: Polynomial Functions (Cont)</b> -Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function. -Use graphing technology to identify the number of and approximate the real zeros of	<b>Unit 3: Powers and Roots</b> -Evaluate $n$ th roots of real numbers using both radical and rational exponent notation. -Apply the properties of exponents to simplify and evaluate expressions containing rational exponents.	<b>Unit 8: Data Analysis and Statistics</b> -Use counting principles and permutations -Use combinations and binomial theorem -Find probabilities of events -Study and interpret probability distributions -Study normal distribution

	<p>a polynomial function.</p> <ul style="list-style-type: none"> <li>-Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function.</li> <li>-Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph.</li> <li>-Use polynomial equations and functions to model and solve real-life problems.</li> </ul>	<ul style="list-style-type: none"> <li>-Perform operations with functions, including those that contain powers.</li> <li>-Find the inverse of both linear and nonlinear functions.</li> <li>-Graph square root and cube root functions.</li> <li>-Solve equations that contain radicals and rational exponents.</li> <li>-Use radical and <math>n</math>th root equations and functions to model and solve real-life problems.</li> </ul>	<ul style="list-style-type: none"> <li>- Select and draw conclusions from samples</li> <li>-Determine the margin of error for a survey</li> <li>-Compare surveys, experiments, and observational studies.</li> </ul>
<b>March</b>	<p><b>Unit 3: Powers and Roots</b></p> <ul style="list-style-type: none"> <li>-Evaluate <math>n</math>th roots of real numbers using both radical and rational exponent notation.</li> <li>-Apply the properties of exponents to simplify and evaluate expressions containing rational exponents.</li> <li>-Graph square root and cube root functions.</li> <li>-Solve equations that contain radicals and rational exponents.</li> </ul>	<p><b>Unit 5: Rational Functions</b></p> <ul style="list-style-type: none"> <li>-Graph rational functions.</li> <li>-Write and use equations for inverse and joint variations.</li> <li>-Multiply and divide rational expressions.</li> <li>-Add and subtract rational expressions.</li> <li>-Simplify complex fractions.</li> <li>-Solve rational equations. Use rational equations and functions to model and solve real-life problems.</li> </ul>	<p><b>Unit 7: Sequences and Series</b></p> <ul style="list-style-type: none"> <li>-Write rules for arithmetic sequences and find sums of arithmetic series.</li> <li>-Write rules for geometric sequences and find sums of geometric series.</li> <li>-Find sums of infinite geometric series.</li> <li>-Use summation notation to write a series.</li> <li>-Evaluate and write recursive rules for sequences.</li> <li>-Use sequences and series to solve real-life problems.</li> </ul>
<b>April</b>	<p><b>Unit 4 Part 1: Exponential Functions</b></p> <ul style="list-style-type: none"> <li>-Graph exponential functions, including those that model growth and decay.</li> <li>-Graph exponential functions that contain the natural base <math>e</math>.</li> <li>-Evaluate and simplify exponential expressions with</li> </ul>	<p><b>Unit 4: Exponentials and Logarithmic Functions</b></p> <ul style="list-style-type: none"> <li>-Graph exponential functions, including those that model growth and decay.</li> <li>-Graph exponential functions that contain the natural base <math>e</math>.</li> <li>-Evaluate and simplify</li> </ul>	<p><b>Unit 6: Conic Sections</b></p> <ul style="list-style-type: none"> <li>-Determine the distance between two points.</li> <li>-Determine the midpoint of the line segment joining two points.</li> <li>-Graph and write equations of parabolas.</li> <li>-Graph and write equations of circles.</li> </ul>

	<p>base <math>e</math>.</p>	<p>exponential expressions with base <math>e</math>.</p> <ul style="list-style-type: none"> <li>-Evaluate and simplify logarithmic expressions.</li> <li>-Graph logarithmic functions.</li> <li>-Use the properties of logarithms.</li> <li>-Solve exponential and logarithmic equations.</li> <li>-Use exponential and logarithmic functions to model and solve real-life problems.</li> </ul>	<ul style="list-style-type: none"> <li>-Graph and write equations of ellipses.</li> <li>-Graph and write equations of hyperbolas.</li> <li>-Classify a conic section using its equation.</li> <li>-Solve quadratic systems algebraically and sketch the graph.</li> <li>-Use conics to model and solve real-life problems.</li> </ul>
<p><b>May</b></p>	<p><b>Unit 4 Part 2: Logarithmic Functions</b></p> <ul style="list-style-type: none"> <li>-Evaluate and simplify logarithmic expressions.</li> <li>-Graph logarithmic functions.</li> <li>-Use the properties of logarithms.</li> <li>-Solve exponential and logarithmic equations.</li> <li>-Use exponential and logarithmic functions to model and solve real-life problems.</li> </ul>	<p><b>Unit 6: Conic Sections</b></p> <ul style="list-style-type: none"> <li>-Determine the distance between two points.</li> <li>-Determine the midpoint of the line segment joining two points.</li> <li>-Graph and write equations of parabolas.</li> <li>-Graph and write equations of circles.</li> <li>-Graph and write equations of ellipses.</li> <li>-Graph and write equations of hyperbolas.</li> <li>-Classify a conic section using its equation.</li> <li>-Solve quadratic systems algebraically and sketch the graph.</li> <li>-Use conics to model and solve real-life problems.</li> </ul>	<p><b>Unit 9: Angles &amp; Trigonometric Ratios</b></p> <ul style="list-style-type: none"> <li>-Describe and measure angles using radian and degree measure.</li> <li>-Convert degree measure to radian measure and vice versa.</li> <li>-Evaluate trigonometric functions of any angle, given a point on its terminal side.</li> <li>-Evaluate trigonometric functions of any angle, with or without using a reference angle.</li> <li>-Identify trigonometric functions of any angle using the period of the function, (or by identifying and using coterminal angles.)</li> <li>-Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions.</li> <li>-Identify trigonometric functions of special angles with or without the use of a unit circle.</li> <li>-Identify and construct a unit</li> </ul>

			<p>circle and recognize its relationship to real numbers.</p> <ul style="list-style-type: none"> <li>-Use a calculator to approximately evaluate a trigonometric function of any angle.</li> <li>-Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles.</li> <li>-Use the Law of Sines to solve oblique triangles.</li> <li>-Use the Law of Cosines to solve oblique triangles.</li> <li>-Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.</li> </ul>
<p><b>June</b></p>	<p><b>Unit 5: Rational Functions</b></p> <ul style="list-style-type: none"> <li>-Graph rational functions.</li> <li>-Write and use equations for inverse and joint variations.</li> <li>-Multiply and divide rational expressions.</li> <li>-Add and subtract rational expressions.</li> <li>-Simplify complex fractions.</li> <li>-Solve rational equations.</li> </ul>	<p><b>Unit 7: Sequences and Series</b></p> <ul style="list-style-type: none"> <li>-Write rules for arithmetic sequences and find sums of arithmetic series.</li> <li>-Write rules for geometric sequences and find sums of geometric series.</li> <li>-Find sums of infinite geometric series.</li> <li>-Use summation notation to write a series.</li> <li>-Evaluate and write recursive rules for sequences.</li> <li>-Use sequences and series to solve real-life problems.</li> </ul>	<p><b>Unit 10: Trigonometric Graphs, Identities, and Equations</b></p> <ul style="list-style-type: none"> <li>-Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant.</li> <li>-Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph.</li> <li>-Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts).</li> <li>-Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or</li> </ul>



			<p>rewrite trigonometric expressions.</p> <ul style="list-style-type: none"> <li>-Use the Fundamental Trigonometric Identities to verify other trigonometric identities.</li> <li>-Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions.</li> <li>-Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions.</li> <li>-Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.</li> </ul>
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Unit 1
<b>Quadratic Functions</b>
Summary and Rationale

Unit 1 extends the study of functions from the first year of algebra to specifically quadratic functions. Students will thoroughly analyze the characteristics and properties of quadratic functions, their equations and graphs. Students will learn different solving and graphing methods, as well as how to write and obtain important information from quadratic equations in different forms. During this unit, it is crucial for students to see the relationship between the *solutions* of a quadratic equation, the *zeros* of a quadratic function, and the *x-intercept* of this function's graph. This connection will follow through with other types of functions that will be studied in future units. The graph of a quadratic function is called a parabola, which is one of the four conic sections that will be studied in Unit 10. Students will also solve and graph one and two variable quadratic inequalities

### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

### Standards

#### Number and Quantity

N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
N.CN.1	Know there is a complex number $i$ such that $i^2 = -1$ , and every complex number has the form $a + bi$ with $a$ and $b$ real.
N.CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
N.CN.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.
N.CN.7	Solve quadratic equations with real coefficients that have complex solutions
N.CN.8	Extend polynomial identities to the complex numbers
N.CN.9	Know the fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

#### Algebra

A.SSE.1	<p>Interpret expressions that represent a quantity in terms of its context.</p> <p>k. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>l. Interpret complicated expressions by viewing one or more of their parts as a single entity.</p>
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	<p>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>g. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>h. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>i. Use the properties of exponents to transform expressions for exponential functions.</p>
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
A.REI.4	<p>Solve quadratic equations in one variable.</p> <p>c. Use the method of completing the square to transform any quadratic equations in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.</p> <p>d. Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equations. Recognize when the quadratic formula gives complex solutions and write them as <math>a + bi</math> and <math>a - bi</math> for real numbers <math>a</math> and <math>b</math>.</p>
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .

F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F-BF-3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them
Geometry	
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)

Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of a collaborative environment
Instructional Focus	
<b>Enduring Understandings:</b>	<b>Essential Questions:</b>

A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.

Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

Quadratic equations are of degree two, therefore they have two solutions that can be determined from a graph (The solutions are the x-intercepts.), or by using different algebraic methods. Four algebraic methods are factoring, taking the square root of both sides, completing the square, or using the Quadratic Formula. The best method to use depends on the forms and characteristics of the given equations, the nature of the solutions, and how precise the solutions should be.

Quadratics can have real number solutions, but can also have solutions in a larger system, called the complex numbers. There are differences between the results of operations on complex numbers from those obtained within the real number system.

The graph of a quadratic function in a coordinate plane is a u-shaped graph, called a parabola. Using algebraic methods to manipulate and/or solve the equation of a quadratic function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

What does the equation and graph of a quadratic function look like?

What is the reason for having the different methods for solving quadratics and when can using one method be necessary or more appropriate than another?

How does using the different forms of a quadratic function help us graph?

What are the types and the differences between the two functions and how are the values of  $a$ ,  $h$ , and  $k$  used to describe their graphs?

$$y = a(x - h) + k$$

$$y = a|x - h| + k$$

$$y = a(x - h)^2 + k$$

How do the results of operations on real numbers differ when applied to the complex numbers?

What are the connections between the algebraic and graphical representations of both one-variable and two variable quadratic inequalities?

The *solutions* of a quadratic equation  $ax^2 + bx + c = 0$  are equal to the *real zeros* of the related quadratic function  $y = ax^2 + bx + c$ , and the *x-intercepts* of this function's graph. Zeros that are not real are not visible on the graph of a function in a coordinate plane of real numbers.

Quadratic inequalities can be one variable or two, and both types can be solved and represented algebraically and graphically.

**Evidence of Learning (Assessments)**

- Tests
- Quizzes
- Homework
- Class participation

**Objectives (SLO)**

Students will know:

- Quadratics
- Quadratic equations
- Inequalities
- Graphing

Students will be able to:

- Graph quadratic functions.
- Factor quadratic expressions and solve quadratic equations by factoring.
- Find the zeros of a quadratic function by factoring.
- -Solve quadratic equations by taking the square root of both sides.
- Perform operations on complex numbers.
- Solve quadratic equations by completing the square.
- Use completing the square to write quadratic functions in vertex form  $y = a(x - h)^2 + k$ .
- Solve quadratic equations using the quadratic formula.
- Find the discriminant of a quadratic equation and use the value to describe the nature of the solutions and to choose the best method for solving.
- Write and graph quadratic functions in standard form ( $y = ax^2 + bx + c$ ), vertex form ( $y = a(x - h)^2 + k$ ), and intercept form ( $y = a(x - p)(x - q)$ ).
- Solve quadratic inequalities in one variable by graphing and algebraically.
- Graph quadratic inequalities in two variables to show all solutions.

·Use quadratic equations, inequalities, and functions to model and solve real-life problems.

### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

### Tier 1 Modifications and Accommodations

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

#### **General Modifications for students struggling to learn:**

- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments.

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work



**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Deriving the Quadratic Formula

### Career Readiness, Life Literacies, and Key Skills NJSLS

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 2

### Polynomial Functions

#### Summary and Rationale

Unit 2 extends the study of functions from the first year of algebra to specifically polynomial functions. Students will first use the Properties of Exponents to simplify and perform operations involving powers, monomials and polynomials. Students will then thoroughly analyze the characteristics and properties of polynomial functions, their equations and graphs. The quadratic functions studied in Unit 5 are polynomial functions of degree two. Students will learn different solving and graphing methods, as well as how to write and obtain important information from polynomial equations of degree greater than two in different forms. During this unit, it is crucial for students to revisit the relationship between the *solutions* of a polynomial equation, the *zeros* of a polynomial function, and the *x-intercept* of this function's graph.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Number and Quantity

N.CN.8	Extend polynomial identities to the complex numbers.
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N.CN.9	Know the fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
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##### Algebra

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. m. Interpret parts of an expression, such as terms, factors, and coefficients. n. Interpret complicated expressions by viewing one or more of their parts as a single entity.
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A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
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A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A.APR.2	Know and apply the remainder Theorem: For a polynomial $p(x)$ and a number $a$ , the remainder of division by $x - a$ is $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$
A.APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomials.
A.APR.4	Prove polynomial identities and use them to describe numerical relationships.
A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.

F.IF.7	<p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude</p>
F.IF.8	<p>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions.</p> <p>a. use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions.</p>
F.IF.9	<p>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p>
F.BF.1	<p>Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard functions types using arithmetic operations.</p> <p>c. Compose functions</p>
F.BF.3	<p>Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.</p>
<p>Geometry</p>	
G.CO.2	<p>Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)</p>
<p>Interdisciplinary Connections</p>	
RL.10	<p>Read and comprehend complex literary and informational texts independently and proficiently.</p>
W.1	<p>Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.</p>

9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.1.12.EG.1	Review the tax rates on different sources of income and on different types of products and services purchased.
9.1.12.RM.2	Identify types of investments appropriate for different objectives such as liquidity, income, and growth.
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.2.12.CAP.13	Analyze how the economic, social, and political conditions of a time period can affect the labor market.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of a collaborative environment
Instructional Focus	
<b>Enduring Understandings:</b>	<b>Essential Questions:</b>

Performing operations on polynomial expressions requires the accurate application of the properties of exponents and the order of operations, as well as the ability to distinguish between like and unlike terms. The requirements for and the results of addition and subtraction of polynomials differ greatly from those involving multiplication, division, and powers.

A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.

Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

All equations of degree one or higher are defined as polynomial equations. The Fundamental Theorem of Algebra states that the number of solutions to a one-variable polynomial equation is equal to the degree of the polynomial. These solutions can be determined through graphing (The solutions are the x-intercepts.) and using one more previously learned and new algebraic methods and theorems. The best method(s) to use depends on the forms and characteristics of the given equations, the nature of the solutions, and how precise the solutions should be.

Polynomials can have real number solutions, but can also have solutions in a larger system, called the complex numbers. There are differences between the results of operations on complex numbers from those obtained within the real number system.

The graphs of polynomial functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a

What are the requirements for the addition and subtraction of monomials and polynomials? Are these the same for multiplication and division? How do the results of the operations differ?

What steps should be taken when factoring a polynomial expression or equation? What specifics do you look for and how do you know when it is completely factored?

What characteristics, other than its solutions, of a polynomial equation can be used to describe the behavior and sketch the graph of its related function?

What theorems and methods are used to identify the total number of roots and determine the real zeros of a polynomial function?

Which types of roots are seen on the graph of a polynomial function? Which are not visible? What happens in the graph if there is a double root?

What are the types and the differences between the two functions and how are the values of  $a$ ,  $h$ , and  $k$  used to describe their graphs?

- $y = a(x - h) + k$
- $y = a|x - h| + k$
- $y = a(x - h)^2 + k$
- $y = a(x - h)^n + k$

polynomial function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

The *solutions* of a polynomial equation are equal to the *real zeros* of the related polynomial function and the *x-intercepts* of this function's graph. Zeros that are not real are not visible on the graph of a function in a coordinate plane of real numbers.

**Evidence of Learning (Assessments)**

- Tests
- Quizzes
- Homework
- Class participation

**Objectives (SLO)**

Students will know:

- Polynomials
- Fundamental Theorem of Algebra
- Rational Zero Theorem
- Graphing

Students will be able to:

- Apply the properties of exponents to simplify and evaluate expressions involving powers.
- Evaluate a polynomial function.
- Graph a polynomial function.
- Add, subtract, and multiply polynomials.
- Factor polynomial expressions.
- Use factoring methods to solve polynomial equations.
- Divide polynomials using long and synthetic division.
- Relate the results of division to the solutions of a polynomial equation and to The Remainder and Factor Theorems.
- Use the Fundamental Theorem of Algebra to determine the number of zeros of a polynomial function.
- Use graphing technology to identify the number of and approximate the real zeros of a polynomial function.
- Use The Rational Zero Theorem to identify possible rational zeros of a polynomial function.

- Use all of the above to identify all zeros of a polynomial function, real and imaginary, and to sketch a graph.
- Use polynomial equations and functions to model and solve real-life problems.

### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

### Tier 1 Modifications and Accommodations

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

#### **General Modifications for students struggling to learn:**

- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignment

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work



**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: End Behavior of Polynomial Functions, Using the Location Principle to identify Zeros

### Career Readiness, Life Literacies, and Key Skills NJSL

Please select all standards that apply to this unit of study:

Act as a responsible and contributing community members and employee

Attend to financial well-being

Consider the environmental, social and economic impacts of decisions

Demonstrate creativity and innovation

Utilize critical thinking to make sense of problems and persevere in solving them

Model integrity, ethical leadership and effective management

Plan education and career paths aligned to personal goals

Use technology to enhance productivity increase collaboration and communicate effectively

Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 3

### **Powers and Roots**

#### Summary and Rationale

Unit 3 connects and extends the familiar ideas of squares and square roots from Algebra 1 to other exponents and roots, including those that are *not* whole numbers in expressions and equations. The unit also extends the study of functions to radical and root functions.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Number and Quantity

N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

##### Algebra

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. <ol style="list-style-type: none"><li>Interpret parts of an expression, such as terms, factors, and coefficients.</li><li>Interpret complicated expressions by viewing one or more of their parts as a single entity.</li></ol>
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
Functions	
F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.BF.4	Find inverse functions. <ul style="list-style-type: none"> <li>a) Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse.</li> <li>b) Verify by composition that one function is the inverse of another.</li> <li>c) Read values of an inverse function from a graph or table, given that function has an inverse.</li> <li>d) Produce an invertible function from a non-invertible function by restricting the domain.</li> </ul>
Geometry	
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.1.12.PB.6	Describe and calculate interest and fees that are applied to various forms of spending, debt and saving.

9.1.12.RM.2	Identify types of investments appropriate for different objectives such as liquidity, income, and growth.
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of a collaborative environment.

### Instructional Focus

<b>Enduring Understandings:</b>	<b>Essential Questions:</b>
<p>Radical expressions can be rewritten using rational exponents and vice versa.</p> <p>The Properties of Exponents apply to all rational exponents.</p> <p>A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.</p> <p>Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.</p> <p>Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the</p>	<p>How are the graphs of a function and its inverse related?</p> <p>What are the types and the differences between the two functions and how are the values of <math>a</math>, <math>h</math>, and <math>k</math> used to describe their graphs?</p> <ul style="list-style-type: none"> <li>• <math>y = a(x - h) + k</math></li> <li>• <math>y = a x - h  + k</math></li> <li>• <math>y = a(x - h)^2 + k</math></li> <li>• <math>y = a(x - h)^n + k</math></li> <li>• <math>y = \sqrt[n]{(x - h)} + k</math></li> </ul> <p>When and how do you check for extraneous solutions when solving</p>

equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

The graphs of radical functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a radical function can throw light on the function's properties and help visualize the behavior of its graph, which can result in the use of more effective and efficient graphing methods.

Functions can be added, subtracted, multiplied, or divided to form a new function. They can also be combined or repeated to form a new function, called a composite function. The new functions created may have different domains and ranges than their parts.

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line  $y = x$ .

The inverse of a power function of  $n^{th}$  degree is an  $n^{th}$  root function. The  $n^{th}$  degree functions that are classified as even functions require domain restrictions.

The use of inverse operations and the properties of equality can be applied to the solving of radical equations and equations with rational exponents. However, certain procedures may lead to invalid solutions called extraneous solutions (or roots).

equations containing radicals and rational exponents?

### Evidence of Learning (Assessments)

- Tests
- Quizzes
- Homework
- Class participation

### Objectives (SLO)

Students will know:

- Powers
- Roots
- Radicals
- Exponents

Students will be able to:

- Evaluate  $n$ th roots of real numbers using both radical and rational exponent notation.
- Apply the properties of exponents to simplifying and evaluating expressions containing rational exponents.
- Perform operations with functions, including those that contain powers.
- Find the inverse of both linear and nonlinear functions.
- Graph square root and cube root functions.
- Solve equations that contain radicals and rational exponents.
- Use radical and  $n$ th root equations and functions to model and solve real-life problems

#### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

#### **Tier 1 Modifications and Accommodations**

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

#### **General Modifications for students struggling to learn:**

- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time

- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics- Solving Radical Inequalities

### Career Readiness, Life Literacies, and Key Skills NJSLs

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 4

### Exponential and Logarithmic Equations

#### Summary and Rationale

Unit 4 extends the study of functions to exponential and logarithmic functions, which are two important families of functions because they model many real-life situations. Students will analyze the characteristics and properties of these functions, their equations and graphs. Students will explore graphs that contain asymptotes as they draw and recognize patterns within the behavior of exponential and logarithmic functions. Previously learned Properties of Exponents and new Properties of Logarithms will be used to simplify expressions and solve equations.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Number and Quantity

N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
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##### Algebra

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. c. Interpret parts of an expression, such as terms, factors, and coefficients. d. Interpret complicated expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a) Factor a quadratic expression to reveal the zeros of the function it defines.



	<p>b) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c) Use the properties of exponents to transform expression for exponential functions</p>
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	<p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a) Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>c) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>e) Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude</p>

F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. a) Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b) Use the properties of exponents to interpret expressions for exponential functions.
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
F.BF.5	Understand the inverse relationships between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a) Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b) Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c) Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
Geometry	
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points at outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

Career Readiness, Life Literacies and Key Skills	
9.1.12.CDM.2	Compare and contrast the advantages and disadvantages of various types of mortgages.
9.1.12.PB.6	Describe and calculate interest and fees that are applied to various forms of spending, debt and saving.
9.1.12.RM.2	Identify types of investments appropriate for different objectives such as liquidity, income, and growth.
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of a collaborative environment
Instructional Focus	
Enduring Understandings:	Essential Questions:
<p>A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.</p> <p>Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.</p>	<p>What values of <math>b</math> does <math>y = b^x</math> represent exponential growth? Decay?</p> <p>How do you determine the asymptotes of an exponential or logarithmic graph?</p> <p>What are the types and the differences between the two functions and how are</p>

Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

A relationship between two variables or two sets of data is an exponential function if the two variables increase (grow) or decrease (decay) by the same *percent* over equal periods of time.

The inverse of an exponential function is a logarithmic function of the same base. Natural exponential and logarithmic functions are of the natural base,  $e$ . Exponential functions have restrictions on the range, therefore logarithmic functions contain domain restrictions. The graphs of these functions contain asymptotes, which are arbitrary lines that a graph approaches as you move away from the origin.

Every function has an inverse, however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line  $y = x$ .

The graphs of exponential and logarithmic functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of an exponential or logarithmic function can throw light on the function's properties such as its zeros and asymptotes, and can also help visualize the behavior of its graph. These strategies can result in the use of more effective and efficient graphing methods.

The use of inverse operations with the properties of equality, exponents, and logarithms can be applied to the solving of exponential and logarithmic equations. However, since the domains of logarithmic functions contain restrictions, obtaining extraneous solutions is a possibility.

the values of  $a$ ,  $h$ , and  $k$  used to describe their graphs?

- $y = a(x - h) + k$
- $y = a|x - h| + k$
- $y = a(x - h)^2 + k$
- $y = a(x - h)^n + k$
- $y = \sqrt[n]{(x - h)} + k$
- $y = ab^{(x-h)} + k$
- $y = \log_b(x - h) + k$

How can you use inverse functions to graph logarithmic functions?

How is solving a logarithmic equation similar to solving an exponential equation? How is it different?

Why do logarithmic equations sometimes contain extraneous solutions?

## Evidence of Learning (Assessments)

Tests

Quizzes

Homework

Class participation

## Objectives (SLO)

Students will know:

- Exponents
- Logarithms
- Graphing

Students will be able to:

- Graph exponential functions, including those that model growth and decay.
- Graph exponential functions that contain the natural base  $e$ .
- Evaluate and simplify exponential expressions with base  $e$ .
- Evaluate and simplify logarithmic expressions.
- Graph logarithmic functions.
- Use the properties of logarithms.
- Solve exponential and logarithmic equations.
- Use exponential and logarithmic functions to model and solve real-life problems.

## Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

## Tier 1 Modifications and Accommodations

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

### General Modifications for students struggling to learn:

Small group instruction within the classroom

Differentiation through content, process, product, and environment

Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.

Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

Special Education: Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract

- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Solving Exponential and Logarithmic Inequalities

### Career Readiness, Life Literacies, and Key Skills NJSLs

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 5

### Rational Functions

#### Summary and Rationale

Unit 5 extends the study of functions to rational functions, which is the ratio of two polynomial functions. Students will analyze the characteristics and properties of these functions, their equations and graphs. Students will explore functions that contain domain and range restrictions as they draw graphs that contain asymptotes and other forms of discontinuity. Familiar concepts involving simplifying and performing operations on rational numbers will be extended to operations of rational expressions. Strategies for solving equations will be extended and applied to solving rational equations.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Algebra

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. <ul style="list-style-type: none"><li>e. Interpret parts of an expression, such as terms, factors, and coefficients.</li><li>f. Interpret complicated expressions by viewing one or more of their parts as a single entity.</li></ul>
A.SSE.2	Use the structure of an expression to identify ways to rewrite it.
A.APR.6	Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x) + r(x) / b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
Functions	
F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F.IF.4	For a function that models relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. <ul style="list-style-type: none"> <li>f) Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> <li>g) Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</li> <li>h) Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</li> <li>i) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</li> <li>j) Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude</li> </ul>
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the functions. <ul style="list-style-type: none"> <li>c) Use the process of factoring and completing the square in a quadratic function to</li> </ul>



	<p>show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>d) Use the properties of exponents to interpret expressions for exponential functions.</p>
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
Geometry	
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.

9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environment

### Instructional Focus

<b>Enduring Understandings:</b>	<b>Essential Questions:</b>
<p>A function is a relationship in which one set of values defines another. All functions can be used to model many important phenomena.</p> <p>Determining an output value of a function, given an input value, requires evaluating the algebraic expression that is being used to represent the function.</p> <p>Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.</p> <p>A rational function is the ratio of two polynomial functions. These functions contain restrictions on their domains and/or ranges. Therefore, their graphs contain asymptotes, holes, and/or discontinuity.</p> <p>The graphs of rational functions in a coordinate plane vary, yet yield various patterns. Using algebraic methods to manipulate and/or solve the equation of a rational function can throw light on the function's properties such as its zeroes, asymptotes, domain, range, and discontinuity, which all can also help visualize the behavior of its graph. These strategies can result in the use of more effective and efficient graphing methods.</p> <p>The previously learned procedures for multiplying, dividing, simplifying, adding, and subtracting numerical</p>	<p>How can you tell whether a set of data pairs (x, y) shows inverse variation?</p> <p>How do you determine the vertical and horizontal asymptotes for the graph of a rational function?</p> <p>What are the types and the differences between the two functions and how are the values of <math>a</math>, <math>h</math>, and <math>k</math> used to describe their graphs?</p> <ul style="list-style-type: none"> <li>● <math>y = a(x - h) + k</math></li> <li>● <math>y = a x - h  + k</math></li> <li>● <math>y = a(x - h)^2 + k</math></li> <li>● <math>y = a(x - h)^n + k</math></li> <li>● <math>y = \sqrt[n]{(x - h)} + k</math></li> <li>● <math>y = ab^{(x-h)} + k</math></li> <li>● <math>y = \log_b(x - h) + k</math></li> <li>● <math>y = \frac{a}{(x-h)} + k</math></li> </ul> <p>How is adding (or subtracting, multiplying, dividing simplifying) rational expressions similar to adding (or subtracting, multiplying, dividing, simplifying) numerical fractions?</p>

fractions are extended to performing the same operations on algebraic rational expressions.

The use of inverse operations, the properties of equalities, and cross multiplication can be applied to the solving of rational equations. Any solutions obtained that are not within the domain of the related function are extraneous.

### Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

### Objectives (SLO)

Students will know:

- Rational equations
- Functions
- Inverse variations

Students will be able to:

- Graph rational functions.
- Write and use equations for inverse and joint variations.
- Multiply and divide rational expressions.
- Add and subtract rational expressions.
- Simplify complex fractions.
- Solve rational equations.
- Use rational equations and functions to model and solve real-life problems.

### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

### Tier 1 Modifications and Accommodations

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

**General Modifications for students struggling to learn:**

Small group instruction within the classroom

Differentiation through content, process, product, and environment

Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.

Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

-Use concrete examples of concepts before teaching the abstract

-Reduce the number of concepts presented at one time

-Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling

-Use of aids (calculator, computer, etc.)

-Frequently check on progress of independent work

-Provide study guides and copy of notes

-Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language

- Monitor the student's comprehension of language used during instruction

- Give written directions to supplement verbal directions

- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

-Teacher will review, restate and repeat directions, as needed

-Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Closure of Polynomial and Rational Expressions, Solving Rational Inequalities

**Career Readiness, Life Literacies, and Key Skills NJSLs**

Please select all standards that apply to this unit of study:

Act as a responsible and contributing community members and employee

Attend to financial well-being

Consider the environmental, social and economic impacts of decisions

Demonstrate creativity and innovation

Utilize critical thinking to make sense of problems and persevere in solving them

Model integrity, ethical leadership and effective management

Plan education and career paths aligned to personal goals

Use technology to enhance productivity increase collaboration and communicate effectively

Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>



## Unit 6

### Conic Sections

#### Summary and Rationale

Unit 6 involves the study of the four conic sections, parabolas, circles, ellipses, and hyperbolas. Conic sections are an important part of the study of algebra and geometry because they have many different real-life applications.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Number and Quantity

N.Q.1

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

##### Algebra

A.SSE.1

Interpret expressions that represent a quantity in terms of its context.

- a) Interpret parts of an expression, such as terms, factors, and coefficients.
- b) Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2

Use the structure of an expression to identify ways to rewrite it.

A.SSE.3

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- d) Factor a quadratic expression to reveal the zeros of the function it defines.
- e) Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- f) Use the properties of exponents to transform expression for exponential functions

A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Geometry	
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)
G.GPE.1	Derive the equations of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and the radius of a circle given by an example.
G.GPE.2	Derive the equation of a parabola given a focus and directrix.
G.GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of the distances from the foci is constant.
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
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### Instructional Focus

<b>Enduring Understandings:</b>	<b>Essential Questions:</b>
<p>The Distance Formula is an application of the previously learned Pythagorean Theorem.</p> <p>A conic section is a graph of an equation of the form <math>Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0</math></p> <p>These relations are studied in algebra because they have various real-life applications.</p> <p>The only conic sections that are functions are parabolas that open upward or downward, previously learned as quadratic functions and hyperbolas that are written in the form of a rational function.</p> <p>Using algebra to manipulate the equation of a conic section, particularly the method of “completing the square” can be used to determine the parts and properties of its graph, and can result in the use of more effective and efficient graphing methods.</p> <p>Two quadratic equations form a quadratic system. The solutions of this type of system are represented by the set of ordered pairs that satisfy both equations in the system.</p> <p>The solutions of these systems can be determined and represented both algebraically and graphically, using the same algebraic methods previously used to solve linear systems. However, solving quadratic systems requires methods learned for solving quadratic equations, and may yield more solutions.</p>	<p>Given the equation of a conic section in the following form <math>Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0</math> how do you classify its graph? How do you rewrite the equation so that you can obtain the information needed to draw its graph? Which conic sections are functions? Which are relations, but not functions?</p> <p>How many solutions are possible for a system containing a conic section and a line? For a system of two conic sections? What would each possible solution case look like graphically?</p> <p>What algebraic methods can be used to solve a system of equations that contains one or more quadratic relations?</p>



## Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

## Objectives (SLO)

Students will know:

- Conic sections
- Graphing

Students will be able to:

- Determine the distance between two points.
- Determine the midpoint of the line segment joining two points.
- Graph and write equations of parabolas.
- Graph and write equations of circles.
- Graph and write equations of ellipses.
- Graph and write equations of hyperbolas.
- Classify a conic section using its equation.
- Solve systems of quadratic systems algebraically and sketch the graph.
- Use conics to model and solve real-life problems.

## Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
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- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

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- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Eccentricity of Conic Sections

### Career Readiness, Life Literacies, and Key Skills NJSLs

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 7

### Sequences and Series

#### Summary and Rationale

Students have studied number patterns (or sequences) since elementary school. Unit 7 will connect this familiar exploration of number patterns to algebra as students write and use rules for sequences and series. An arithmetic sequence has a common difference, so it is similar to the linear functions. A geometric sequence has a common ratio, so it is similar to the exponential functions.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Algebra

A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
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##### Functions

F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
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F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between two forms.
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##### Interdisciplinary Connections

RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
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W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
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9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.1.12.EG.1	Review the tax rates on different sources of income and on different types of products and services purchased.
9.1.12.PB.6	Describe and calculate interest and fees that are applied to various forms of spending, debt and saving.
9.1.12.RM.6	Identify types of investments appropriate for different objectives such as liquidity, income, and growth.
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environment
Instructional Focus	
<b>Enduring Understandings:</b>	<b>Essential Questions:</b>

Algebra can be used to write and use rules for number patterns that exist in sequences and series. A sequence is a list of terms that demonstrate a number pattern, while a series is an expression formed by adding the terms of the sequence. Both can be either finite or infinite.

An arithmetic sequence has a common difference, so it is similar to a linear function. A geometric sequence has a common ratio, so it is similar to an exponential function.

A recursive sequence is one that uses one or more previous term(s) to obtain the next term.

What is the difference between a sequence and a series?

What makes a sequence or series arithmetic? geometric?

What information do you need to find a term of an arithmetic (or geometric) series?

How do you know if a sum of an infinite geometric series exists? And if it does, what information is needed to determine the sum?

How can summation (or sigma) notation be used to represent a series?

### Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

### Objectives (SLO)

Students will know:

- Sequences
- Series

Students will be able to:

- Write rules for arithmetic sequences and find sums of arithmetic series.
- Write rules for geometric sequences and find sums of geometric series.
- Find sums of infinite geometric series.
- Use summation notation to write a series.
- Evaluate and write recursive rules for sequences.
- Use sequences and series to solve real-life problems.

Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

### **Tier 1 Modifications and Accommodations**

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

#### **General Modifications for students struggling to learn:**

Small group instruction within the classroom

Differentiation through content, process, product, and environment

Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.

Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Translating Between Recursive and Explicit Rules for Sequences

Career Readiness, Life Literacies, and Key Skills NJSL

Please select all standards that apply to this unit of study:

Act as a responsible and contributing community members and employee

Attend to financial well-being

Consider the environmental, social and economic impacts of decisions

Demonstrate creativity and innovation

Utilize critical thinking to make sense of problems and persevere in solving them

Model integrity, ethical leadership and effective management

Plan education and career paths aligned to personal goals

Use technology to enhance productivity increase collaboration and communicate effectively

Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

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## Unit 8

### Data Analysis and Statistics

#### Summary and Rationale

Unit 6 will begin with the exploration combinations and using them to calculate probabilities. Students will learn about probability distributions and analyze normal distributions using statistics. They will also learn new appropriate sampling methods for collecting data when conducting surveys and experiments. These skills will help students appropriately conduct surveys, calculate and report statistical measures when analyzing data, and form a solid foundation for a course study of statistics.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Algebra

A.APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
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A-APR-5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$ , where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)
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##### Statistics and Probability

S-ID-1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
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S-ID-2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
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S.ID-3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
S.ID-4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
S-IC-1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
S-IC-3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
S-IC-4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
S-IC-5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
S-IC-6	Evaluate reports based on data.
S-MD-3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	

9.1.12.CFR.4	Demonstrate an understanding of the interrelationships among attitudes, assumptions, and patterns of behavior regarding money, saving, investing, and work across cultures.
9.1.12.FP.5	Evaluate how behavioral bias (e.g., overconfidence, confirmation, recency, loss aversion, etc.) affects decision-making.
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.IML.3	Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions .
9.4.12.IML.4	Assess and critique the appropriateness and impact of existing data visualizations for an intended audience.
9.4.12.IML.8	Evaluate media sources for point of view, bias, and motivations.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.2	Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of a collaborative environment.
<b>Instructional Focus</b>	
<b>Enduring Understandings:</b>	<b>Essential Questions:</b>
A combination is a selection of $r$ objects from a group of $n$ objects, where the order is not important.	What is a combination?  How are combinations used to calculate probabilities?

Pascal's triangle can be used to find combinations and to expand binomials.

The Binomial Theorem can be used to expand binomials or to find a coefficient of an expansion.

A random variable is a variable whose value is determined by the outcomes of a random event.

A probability distribution is a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

A binomial experiment is an experiment consisting of a finite number of trials where each trial has only two possible outcomes: success and failure. The probability of success is the same for each trial.

One type of probability distribution is a normal distribution, which is modeled by a bell-shaped curve called a normal curve that is symmetric about the mean. The total area under a normal curve is 1. Standard deviations are used to describe how far above or below data is from the mean.

The standard normal distribution has a mean of 0 and a standard deviation of 1.

The z-score for a particular x-value in a normal distribution is the number of standard deviations the x-value lies above or below the mean.

A population is a group of people or objects that you want information about. When it is too difficult, time consuming, or expensive to survey everyone in a population, information is gathered from a sample.

How can Pascal's triangle help you expand a binomial?

What does it mean if data is normally distributed?

How are the mean and standard deviation used to analyze normal distributions?

What are the types of samples that can be collected for an experiment?

What makes a survey question biased?

The margin of error gives a limit on how much the responses of a sample would differ from the responses of the entire population.

Survey questions that are flawed in a way that leads to inaccurate results are called biased questions.

An experiment imposes a treatment on individuals in order to collect data on their response to the treatment.

An observational study observes individuals and measures variables without controlling the individuals or their environment,

### Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

### Objectives (SLO)

Students will know:

- Combinations
- Pascal's Triangle
- Binomial theorem
- Random sample
- Binomial distribution
- Binomial experiment
- Symmetric
- Skewed
- Normal distribution
- Normal curve
- Standard normal distribution
- z-score
- Population
- Sample
- Unbiased sample
- Unbiased sample
- Margin of error
- Biased questions experiment

Students will be able to:-

- Find combinations
- Solves problems involving combinations
- Use Pascal's triangle to expand a binomial
- Use the Binomial Theorem
- Construct and interpret a probability distribution
- Construct and interpret a binomial distribution
- Classify a distribution as symmetric or skewed
- Find a normal probability
- Interpret normally distributed data
- Classify samples
- Identify a biased and unbiased sample
- Find a margin of error of a sample
- Select and draw conclusions from samples
- Determine the margin of error for a survey
- Identify and correct survey questioning.
- Identify experiments and observational studies
- Evaluate a published report
- Design an experiment or observational study.

- Observational study

### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

### Tier 1 Modifications and Accommodations

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

#### General Modifications for students struggling to learn:

- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Calculating the Area under a Normal Curve

### Career Readiness, Life Literacies, and Key Skills NJSL

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 9

### Angles and Trigonometric Ratios

#### Summary and Rationale

Unit 9 continues the study of trigonometry, with a specific focus on the study of angles and triangles. Trigonometry is closely tied to both algebra and geometry. Students began their study of trigonometry in their geometry courses with right triangles. This unit will provide opportunities for students to further explore the more complex and comprehensive ideas behind the use of trigonometry in triangle measurement. Students will find trigonometric functions of angles, solve triangles, and use trigonometry to calculate the area of triangles.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Functions

F.TF.1	Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$ , $\frac{\pi}{4}$ , and $\frac{\pi}{6}$ and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$ , $\pi + x$ and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.
I.TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

##### Geometry

G.CO.1	Know precise definitions of angles, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance
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	around a circular arc.
G.SRT.7	Explain and use the relationship between the side and cosine complementary angles
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
G.SRT.9	Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10	Prove the Laws of Sines and Cosines and use them to solve problems
G.SRT.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.



9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environment

Instructional Focus

<b>Enduring Understandings:</b>	<b>Essential Questions:</b>
<p>Trigonometric functions of acute angles are defined by ratios of side lengths of right triangles. These ratios are: sine, cosine, tangent, cosecant, secant, and cotangent.</p> <p>The angles <math>30^\circ</math>, <math>45^\circ</math>, and <math>60^\circ</math> occur frequently in trigonometry. These are the angles of the two special right triangles previously learned in geometry (<math>30^\circ</math>-<math>60^\circ</math>-<math>90^\circ</math> and <math>45^\circ</math>-<math>45^\circ</math>-<math>90^\circ</math>) The values of the six trigonometric functions of these angles do not need to be memorized since they can quickly be generated by drawing the two special right triangles and labeling the sides of each.</p> <p>Trigonometric functions and their inverses can be used to find a missing side length or angle measure in a right triangle. Finding all missing lengths and angle measures is called solving a right triangle.</p> <p>An angle is formed by two rays that have a common endpoint, called the vertex. You can generate an angle by fixing one ray, called the initial side, and rotating the other ray, called the terminal side, about the vertex. In a coordinate plane, an angle whose vertex is at the origin and whose initial side is the positive x-axis in standard position. The phrase “the terminal side of <math>\theta</math> lies in quadrant...” is simply saying that <math>\theta</math> lies in that quadrant. Two angles in standard position are coterminal if their terminal sides coincide. An angle</p>	<p>How do you convert between degrees and radians?</p> <p>How can you use the trigonometric ratios to find the measurements of sides or angles of a right triangle?</p> <p>How can you use The Law of Sines and The Law of Cosines to solve any triangle?</p>

coterminal with a given angle can be found by adding or subtracting multiples of  $360^\circ$ .

Angles can be measured in degrees and in radians. The measure of an angle is determined by the amount and direction of a rotation from the initial side to the terminal side. The angle measure is positive if the rotation is counterclockwise and negative if the rotation is clockwise. The terminal side of an angle can make more than one complete rotation.

To define a radian, consider a circle with radius  $r$  centered at the origin. One radian is the measure of an angle in standard position whose terminal side intercepts the arc of length  $r$ .

Since the circumference of a circle is equal to  $2\pi r$ , there are  $2\pi$  radians in a circle. Therefore  $2\pi$  radians is equivalent to  $360^\circ$ . ( $\pi$  radians =  $180^\circ$ , and so on...)

The length of an arc and the area of a sector of a circle can be calculated using the central angle of the circle in degrees (previously learned), and in radians.

Reference angles are used to evaluate trigonometric functions of *any* angle.

The Law of Sines and The Law of Cosines can be used to solve *any* triangle. (Determine all missing side lengths of angle measurements.) The use of one law rather than the other depends on the known information about the triangle. Some problems may show cases in which a triangle cannot be made under the given conditions (No solution). Other cases yield one or two possible triangle solutions.

Trigonometric ratios can also be used to calculate the area of *any* triangle.

### Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

**Objectives (SLO)**

Students will know:

- Trigonometric functions
- Law of Sines
- Law of Cosines
- Heron's Area Formula

Students will be able to:

- Describe and measure angles using radian and degree measure.
- Convert degree measure to radian measure and vice versa.
- Evaluate trigonometric functions of any angle, given a point on its terminal side.
- Evaluate trigonometric functions of any angle, with or without using a reference angle.
- Identify trigonometric functions of any angle using the period of the function, (or by identifying and using coterminal angles.)
- Use the Fundamental Trigonometric Identities to determine the values of all six trigonometric functions.
- Identify trigonometric functions of special angles with or without the use of a unit circle.
- Identify and construct a unit circle and recognize its relationship to real numbers.
- Use a calculator to approximately evaluate a trigonometric function of any angle.
- Use the right triangle trigonometric ratios and inverse trigonometric functions and Pythagorean Theorem to solve right triangles.
- Use the Law of Sines to solve oblique triangles.
- Use the Law of Cosines to solve oblique triangles.
- Use the Law of Sines and/or Heron's Area Formula to calculate the area of an oblique triangle.

**Suggested Resources/Technology Tools**

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified

- Google Classroom
- Desmos graphing calculator
- Kuta Software

### **Tier 1 Modifications and Accommodations**

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- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time
- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topic Trigonometric Functions on the Unit Circle

### **Career Readiness, Life Literacies, and Key Skills NJSL**

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation

Utilize critical thinking to make sense of problems and persevere in solving them

Model integrity, ethical leadership and effective management

Plan education and career paths aligned to personal goals

Use technology to enhance productivity increase collaboration and communicate effectively

Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>

## Unit 10

### Trigonometric Graphs, Identities, and Equations

#### Summary and Rationale

The focus of Unit 10 is the study of trigonometric functions of real numbers, used to model periodic behavior. Students will have opportunities to see more connections between trigonometry and algebra as they graph trigonometric functions and their inverses in a coordinate plane. Just as they have done with other types of functions in previous units, students will analyze the characteristics of their equations and graphs. Students will also use advanced algebra skills to evaluate trigonometric functions, verify trigonometric identities, and solve trigonometric equations.

#### Recommended Pacing

For recommended pacing refer to the scope and sequence for each course.

#### Standards

##### Number and Quantity

N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
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##### Algebra

A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
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##### Functions

F.IF.4	For a function that models relationships between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
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F.IF.5	Relate domain of a function to its graph and, where applicable, to the quantitative
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	relationship it describes.
F.IF.6	Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.
F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Write exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
F.TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
F.TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
F.TF.8	Prove the Pythagorean identity $\sin^2(q) + \cos^2(q) = 1$ and use it to find $\sin(q)$ , $\cos(q)$ , or $\tan(q)$ given $\sin(q)$ , $\cos(q)$ , or $\tan(q)$ and the quadrant of the angle.
F.TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
Geometry	
G.CO.1	Know precise definitions of angles, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO.2	Represent transformations in the plane, describe transformations as functions that take points in the plane as inputs and give other points at outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch)
G.SRT.7	Explain and use the relationship between the side and cosine complementary angles.
G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Interdisciplinary Connections	
RL.10	Read and comprehend complex literary and informational texts independently and proficiently.
W.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
9.1	All students will demonstrate the creative, critical thinking, collaboration, and problem solving skills needed to function successfully as both global citizens and workers in diverse ethnic and organizational cultures.
Integration of Technology	
8.1.8.A.5	Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
Career Readiness, Life Literacies and Key Skills	
9.2.12.CAP.3	Investigate how continuing education contributes to one's career and personal growth.
9.2.12.CAP.4	Evaluate different careers and develop various plans.
9.2.12.CAP.8	Determine job entrance criteria (e.g., education credentials, math/writing/reading comprehension tests, drug tests) used by employers in various industry sectors.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
9.4.12.CI.2	Identify career pathways that highlight personal talents, skills, and abilities.
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice.
9.4.12.CT.2	Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
9.4.12.TL.1	Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specific task.
9.4.12.TL.3	Analyze the effectiveness of the process and quality of collaborative environment
Instructional Focus	
<b>Enduring Understandings:</b>	<b>Essential Questions:</b>



A function is a relationship in which one set of values defines another. Each value of the input variable (value in the domain) is associated with a unique value of the output variable (value in the range.) In order to determine if an equation or a set of ordered pairs represents a function, the solutions of the equation or the ordered pairs can be organized in a table or plotted on a graph. If the table of values shows that each value in one set is paired with exactly one value in the other set, the relation is a function. The vertical line test uses the graph to determine whether a relation is a function. All functions can be used to model many important phenomena.

Periodic functions have graphs that have repeating patterns that continue indefinitely. The shortest repeating portion is called a cycle. The horizontal length of each cycle is called the period. Trigonometric functions are periodic functions.

Determining an output value of a trigonometric function, given an input value, requires evaluating the trigonometric expression that is being used to represent the function. Functions can be represented using an equation, or through a graph of the ordered pairs on a coordinate plane that satisfy the equation. The graph of a function is a useful way of visualizing the relationship of the function, as well as its complete domain and range.

The graphs of the sine and cosine are periodic functions that both have the following characteristics: The domain of each function is all real numbers and the range of each function is  $-1 \leq y \leq 1$ . Each graph has a period of  $2\pi$  (or  $360^\circ$ .)

The graph of tangent is a periodic function with a domain of all real numbers except odd multiples of  $\pi/2$ . At odd multiples of  $\pi/2$ , the graph has vertical asymptotes. The range is all real numbers. The graph has a period of  $\pi$  (or  $180^\circ$ .)

Why are trigonometric functions called periodic functions?

What is a trigonometric identity?

How can trigonometric identities be used to determine exact trigonometric values of uncommon angles?

How can you solve trigonometric equations?

Families of functions are all transformed (translated (or shifted), reflected, and/or dilated) in the same ways, as the values of  $a$ ,  $h$ , and  $k$  similarly affect their graphs:

$$y = a(x - h) + k$$

$$y = a|x - h| + k$$

$$y = a(x - h)^2 + k$$

$$y = a(x - h)^n + k$$

$$y = \frac{a}{(x - h)} + k$$

$$y = ab^{(x-h)} + k$$

$$y = \log_b(x - h) + k$$

$$y = a \sin b(x - h) + k$$

$$y = a \cos b(x - h) + k$$

$$y = a \tan b(x - h) + k$$

Every function has an inverse however the inverse is not always another function. There is an algebraic procedure for finding the inverse of a function. The domain of the function is the range of its inverse. The range of a function is the domain of its inverse. (Input and output values switch.) Graphs of inverse functions are reflections about the diagonal line  $y = x$ .

Trigonometric and inverse trigonometric functions and their graphs can be used to model and solve real-world problems, particularly those that involve periodic behavior.

Trigonometric identities are equations that are true for all values of  $x$  in their domain. Examples of Fundamental Trigonometric Identities are: Reciprocal Identities, Tangent and Cotangent Identities, Pythagorean Identities, Cofunction Identities, and Negative Angle Identities. These trigonometric identities can be used to evaluate trigonometric functions, simplify trigonometric expressions, and verify other identities.

A verification of a trigonometric identity is a chain of equivalent expressions showing that one side of the identity is equal to the other side. When verifying an identity a common and useful strategy is to begin with the

expression from one side and manipulate it algebraically until it is identical to the other side.

Properties of real numbers and equality, along with the use of inverse operations and trigonometric identities, can transform a trigonometric equation into one or a series of equivalent simpler equations. These properties can be used repeatedly to isolate the variable. Algebraic techniques such as factoring or The Quadratic Formula may also be necessary during the process of finding solutions to trigonometric equations in quadratic form. Extraneous solutions can result if an equation is being solved within a restricted domain.

Sum, Difference, Double, and Half-Angle Formulas make it possible to determine exact trigonometric values for less common angles using the more common angles such as  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , etc.

### Evidence of Learning (Assessments)

Tests  
Quizzes  
Homework  
Class participation

### Objectives (SLO)

Students will know:

- Trigonometric functions of real numbers
- Formulas
- Graphing

Students will be able to:

- Sketch the graphs, analyze, compare, and identify domains and ranges of the basic trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant.
- Find the amplitude and period of a trigonometric function and use these characteristics to sketch its graph.
- Identify and sketch translations of trigonometric graphs, (vertical shifts and phase shifts).
- Evaluate, graph and identify the domains and ranges of inverse trigonometric functions.
- Write equations for inverse trigonometric functions.

- Evaluate compositions of trigonometric functions.
  - Use trigonometric functions and their inverses to model and solve real-life problems.
  - Use the Fundamental Trigonometric Identities to evaluate trigonometric functions, simplify and/or rewrite trigonometric expressions.
  - Use the Fundamental Trigonometric Identities to verify other trigonometric identities.
  - Use the Sum and Difference Formulas (or identities) to evaluate exact values for trigonometric functions.
  - Use the Double and Half-Angle Formulas (or identities) to evaluate exact values for trigonometric functions.
- Use standard algebraic techniques, in addition to the objectives above, to solve trigonometric equations.

#### Suggested Resources/Technology Tools

- Textbooks, workbooks, and assessment aides
- Online textbook
- NJSLA Practice Tests and Released Items
- Khan Academy; [www.insidemathematics.org/performanceassessment-tasks](http://www.insidemathematics.org/performanceassessment-tasks)
- Calculators when specified
- Google Classroom
- Desmos graphing calculator
- Kuta Software

#### **Tier 1 Modifications and Accommodations**

*Including special education students, Multilingual Language Learners (MLLs), students at risk of school failure, gifted and talented students, and students with 504 plans;*

##### **General Modifications for students struggling to learn:**

- Small group instruction within the classroom
- Differentiation through content, process, product, and environment
- Individual feedback and praise towards what is done correctly based upon effort, attitude and strategy.
- Help students manage individual stressors for the student and plan alternate pathways for completion of assignments

**Special Education:** Modifications are determined by each student's Individual Education Plan. Examples include:

- Use concrete examples of concepts before teaching the abstract
- Reduce the number of concepts presented at one time

- Give additional presentations by varying the methods using repetition, simpler explanations, more examples and modeling
- Use of aids (calculator, computer, etc.)
- Frequently check on progress of independent work
- Provide study guides and copy of notes
- Provide repetition and practice

**MLL:** Modifications are determined by each student. Examples include:

- Provide students with notes, examples, tests, and quizzes in their primary language
- Monitor the student's comprehension of language used during instruction
- Give written directions to supplement verbal directions
- Frequently check on progress of independent work

**504:** Modifications are determined by each student's 504 plan. Examples include:

- Teacher will review, restate and repeat directions, as needed
- Frequently check on progress of independent work

**Gifted and Talented** - Extension Topics: Inverse Trigonometric Functions

### Career Readiness, Life Literacies, and Key Skills NJSLs

Please select all standards that apply to this unit of study:

- Act as a responsible and contributing community members and employee
- Attend to financial well-being
- Consider the environmental, social and economic impacts of decisions
- Demonstrate creativity and innovation
- Utilize critical thinking to make sense of problems and persevere in solving them
- Model integrity, ethical leadership and effective management
- Plan education and career paths aligned to personal goals
- Use technology to enhance productivity increase collaboration and communicate effectively
- Work productively in teams while using cultural/global competence

Suggestions on integrating these standards can be found at: <https://www.nj.gov/education/standards/clicks/>